THE
PRESCRIBING OF SPECTACLES

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BY

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ILLUSTRATED WITH DIAGRAMS

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PREFACE TO THIRD EDITION.

THE greater part of this book has been rewritten as some valuable work has been done on this subject since the last edition was published.

Professor Gullstrand, of Upsala, and Professor Tscherning, of Copenhagen, after prolonged and careful investigations, have given us new values for the constants of the eye and consequently for the cardinal points of the emmetropic eye. I give the final results of each of these observers on *p.* 151 and *p.* 155, and for reasons given on *p.* 156 I have throughout adopted those of Professor Gullstrand.

For the last thirty years I have studied the subject of periscopic lenses. My first paper was published in Knapp's *Archives*, 1901; the method I now use is explained in detail (*pp.* 200–6) and is better than that given in the last edition of this book. Full tables of my results will be found on *pp.* 207–8. I have also worked out tables (*pp.* 183–4) for those who use the ophthalmometer in determining astigmatism.

Every new optical statement that I have made in the body of the book has been proved in the optical section, I hope clearly and concisely.

I am much indebted to Mr. Alexander MacRae for reading and correcting the proofs.

A. S. PERCIVAL.

Shenley, Woking,
April, 1928.

PREFACE TO FIRST EDITION.

This small work describes the practical methods of determining errors of refraction and errors of muscular balance, and gives, I hope, clear directions for prescribing the appropriate spectacles for their relief. The last fifty pages, comprising the Optical Section, give the mathematical solution of most of the optical problems that arise in dealing with the subject, as well as five tables that are frequently required.

I am deeply indebted to Dr. Duane for providing me with the results of his latest researches on accommodation, and to Dr. Tscherning for his last evaluation of the optical constants of the eye.

My little book, I may add, is not a slavish summary of the practice of others, and some of the methods detailed are original. This, I think, justifies its production.

Archibald Stanley Percival.

17, *Claremont Place, Newcastle-upon-Tyne,*
July, 1910.

CONTENTS.

The Prescribing of Spectacles.

CHAPTER I

INTRODUCTORY—ACCOMMODATION

THE first spectacles of which we have any description were those made by Roger Bacon, towards the end of the thirteenth century. We learn that he ground and polished some glass which he obtained from Belgium, and thus with his own hands made the first convex reading glasses. Concave lenses were invented shortly afterwards.

There is reason, however, to believe that convex lenses were used as magnifying glasses in very early times, for Layard found in the ruins of Nimrud, near Nineveh, a convex lens of rock crystal; indeed, we might have inferred that magnifiers must have been used many years B.C. from the perfection of the carving of ancient gems. Aristophanes, in *The Clouds*, 423 B.C., speaks of a crystal lens used as a burning-glass for lighting fires.*

Astigmatism was first discovered by Thomas Young,

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*ΣΤ. Ἤδη παρὰ τοῖσι φαρμακοπώλαις τὴν λίθον
ταύτην ἑόρακας τὴν καλήν, τὴν διαφανῆ,
ἀφ' ἧς τὸ πῦρ ἅπτουσι; ΣΩ. Τὴν ὕαλον λέγεις;
(ΝΕΦΕΛΑΙ, *ll.* 766–9).

Strepsiades: You know that sort of stone the oddment-dealers sell,
A pretty thing, and you can see through it quite well,
'Tis used for lighting fires?
Socrates: The lens, you mean? (*Rudd's Translation.*)

who, in the *Philosophical Transactions* for 1793, published an account of the asymmetry of his own eye, and attributed it to an oblique position of his lens.

His account is so accurate that we infer his error would have been corrected by −1·7 D cyl. axis 90°, and that it would be induced by an 18° rotation of his lens round the vertical axis. His astigmatism was peculiar as it still persisted when his eye was plunged in water, so it may have been of lenticular origin. (Cf. *p.* 57).

In 1827, the astronomer Airy corrected his defective sight by means of cylindrical lenses, and since that time spectacles for various purposes have been made, so that now they are expected to relieve not only errors of refraction, but also any slight want of balance that may be found in the ocular muscles.

The eye may be regarded as a sort of photographic camera, that produces upon the retina, by its refracting system, an inverted image of the object viewed. It is, moreover, able to focus objects at different distances. Normally, an eye at one moment can see distinctly objects as remote as the stars, and at another moment objects at a distance of ten inches or less. This focusing power of the eye is called its power of accommodation.

ACCOMMODATION.

Accommodation is brought into play by a contraction of the ciliary muscle, which renders the lens of the eye more convex, and therefore increases its power of refraction.

There are two conflicting theories regarding the action of the ciliary muscle. It is known that during accommodation the central part of the anterior surface of the lens becomes more convex. Helmholtz regarded the lens as an elastic body which is flattened by the traction of the zonula; when the ciliary muscle contracts, the choroid is dragged forwards and the tension of the zonula is relaxed and the lens therefore becomes more convex.

Tscherning asserts that in a dead eye the lens assumes the shape of an unaccommodated eye. The accommodated lens shows a maximum curvature at its central part, while it is very much flattened towards its periphery, presenting the appearance of lenticonus. He finds the lens not to be the simple elastic body that Helmholtz suggests, but regards it, in the adult, as composed of two parts ; a firm nucleus of very pronounced curvature surrounded by a soft cortical layer enclosed in a nearly inextensible capsule of much less curvature. This " accommodative layer " as he calls it has the consistence of thick gum ; as age advances, the nucleus increases, while the accommodative layer diminishes and with it the amplitude of accommodation. The ciliary muscle is triangular in section, its external side rests on the sclera and reaches nearly to the canal of Schlemm, while its short anterior side faces towards the anterior chamber and the root of the iris. During contraction the antero-external angle remains fixed, the posterior extremity advances, and we can see in the anterior chamber that the antero-internal angle recedes. Tscherning points out that the recession of the anterior part exerts on the zonula the traction which produces the increased convexity of the anterior surface (due to the pressure of the nucleus) ; the advancement of the posterior extremity draws forward the choroid, and hence sustains the vitreous, and indirectly the lens, so that the latter does not recede from the anterior traction.

Tscherning's view gives a clear explanation of the acquired hypermetropia and the loss of accommodation that ensue with age ; it is not now, however, accepted by many ophthalmic surgeons of repute owing to the recent work that has been done on the subject : (1) The experiments of Hess seemed to show quite conclusively that the zonule was relaxed during accommodation ; (2) Those of Hartridge and Yamada (*Brit. Jour. Ophthalmol.*, 1922) on

the cat confirmed the view of Helmholtz, and they found that if the lenses were removed from the cat's eyes, they were in a state of accommodation. To Tscherning must be given the credit of maintaining the increased curvature of the polar part and the relative flattening of the peripheral part of the lens, in spite of staunch, or even virulent, criticism from his opponents. This appeared to be in direct opposition to the classical theory, until Fincham explained it in a masterly paper in 1924 (*Trans. Optical Soc.*, xxvi). He there drew attention to an observation by Bowman that the thickness of the capsule is not uniform. The capsule of the anterior surface has its maximum thickness in a zone 3 mm. from the pole, whereas the capsule of the posterior surface, which is extremely thin over the greater part, reaches its maximum near the equator of the lens. This entirely accounts for the lenticonus appearance which has been observed on the anterior and occasionally on the posterior surface of the lens.

A detective camera of fixed length, that is adjusted for distance, may be adapted for a distance of three feet by placing in front of its lens a convex glass of 3-ft. focus. Similarly, an eye which, in its state of rest, is adapted for parallel rays may, by exercising its accommodation, adapt itself to a distance of three feet. The accommodation exercised in this case is the power of a lens of focus 3 ft.

For example, a person of 25, who is able to see the stars distinctly when his accommodation is relaxed, will, on exerting his accommodation to the utmost, be able to see distinctly an object only four inches from his eye. In other words, the distance of his far point, or *punctum remotum* (R), is infinite (∞), while the distance of his near point, or *punctum proximum* (P), is 4 in. The amplitude of accommodation here is the power of a lens of 4-in. focus.

The metric system is invariably used in dealing with

problems of refraction. A metre is 39·37 in., or roughly, 40 in., so that 4 in. is $\frac{1}{10}$ metre, 10 centimetres, or 100 millimetres.* A lens of 4-in. focus, or $\frac{1}{10}$ metre, is called a lens of 10 dioptres. The stronger the lens, the shorter is its focal distance. A lens of focal distance 10 cm. is ten times stronger than a lens of focal distance 100 cm. One of focal distance 1 metre, or 100 cm., is called a lens of one dioptre, and when convex is written $+1$ D, i.e. $D = \frac{1}{F'}$ where F' is the first focal distance expressed in metres. A $+4$ D lens means a convex lens, the first focal distance of which is $\frac{1}{4}$ metre or 25 cm., while a -5 D lens is a concave glass, the first focal distance of which is $-\frac{1}{5}$ metre, or -20 cm. It is clear that to express a lens by its power in dioptres is much more convenient than by its focal length.

The *amplitude* of accommodation is the greatest power of accommodation that can be exercised; it is found to decrease as age advances. We have found that the person of 25, referred to above, had an amplitude of accommodation of 10 dioptres, or 10 D. If the person had been 44 years old, it probably would have been found that only with the greatest effort could he have seen distinctly an object 25 cm. (or 10 in.) off. In that case his amplitude would have been indicated by the power of a lens of focal distance 25 cm., i.e., by a $+4$ D lens.

From elementary optics we know that if p denote the distance (considered positive) of the lens from the object, if q denote the distance of the lens from the image, and if F' represent its first focal distance in metres, $\frac{1}{p} - \frac{1}{q} = \frac{1}{F'} =$ its power in dioptres.

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* The following approximate equivalents are useful: 1 in. = 2·54 cm.; 1 ft. = 30·48 cm.; 1 mile = 1·61 Km.

Similarly, we say that the amplitude of accommodation or $A = \frac{1}{P} - \frac{1}{R} = \frac{1}{F'}$, where P is the distance (in metres) of the eye from the *punctum proximum* (*p. p.*), and R that of the eye from the *punctum remotum* (*p. r.*). In fact A is measured by the power in dioptres of the lens which would form an image at P of an object at R.

Suppose, for instance, a myopic patient presents himself who cannot see distinctly at a greater distance than 2 metres, but by exerting his utmost focusing power he can see an object at 12·5 cm., or $\frac{1}{8}$ metre from his eye. In such a case $R = 2$, $P = \frac{1}{8}$, therefore his amplitude is

$$A = \frac{1}{P} - \frac{1}{R} = 8 - \cdot 5 = 7\cdot 5 \text{ D}.$$

Let us take the case of a hypermetrope of $+3$ D. In this case, when accommodation is relaxed, in order that rays may come to a focus on his retina, they must originally be converging to a point $\frac{1}{3}$ metre behind his eye. Consequently his punctum remotum is negative or $R = -\frac{1}{3}$. Now if his punctum proximum is 12·5 cm., or $\frac{1}{8}$ metre distant,

$$A = \frac{1}{P} - \frac{1}{R} = 8 - (-3) = 8 + 3 = 11 \text{ D}.$$

It will be noticed that in these two illustrations P and R denote the distances of the eye from *p. p.* and *p. r.*, but this is very indefinite. Donders estimated the distances of the nodal point of the eye, but it is much more convenient to denote by P and R the distances from *p. p.* and *p. r.* of that position in which the spectacles for the relief of defective accommodation are placed, and this is very nearly the anterior focal plane of the eye. Duane places the spectacles at 14 mm. from the cornea, and he says, "When we speak of a deficiency of accommodation of 6 D we mean one that will be compensated for by a lens of that strength placed at the anterior focus—not at the principal point—of the eye." It should be noted, however, that according to both Tscherning's and Gullstrand's last published tables of the ocular constants the anterior focal plane of the eye is more than 15 mm. in front of the corneal apex.

Influence of Age.—The position of the punctum remotum, or the static refraction of the eye, does not change until after the age of 50, when a slowly progressive "acquired hypermetropia" is found to occur, so that at 80 hypermetropia to the extent of + 2·25 D has developed, as is shown in the following table :—

Age	Acquired Hypermetropia	Age	Acquired Hypermetropia
55	·1 D	70	1·2 D
60	·3 D	75	1·75 D
65	·75 D	80	2·25 D

This "acquired hypermetropia" is partly due to the increased refractive index of the cortical layers of the lens. In youth the lens consists of a dense nucleus surrounded by concentric layers of a less dense material (*Fig.* 1). These peripheral layers may be considered as forming two *diverging* menisci, *a* and *b*, which enclose the nucleus and diminish its refractive power. When the cortical layers of the lens become more dense with age, the refractive power of these menisci becomes greater, and therefore the power of the whole lens becomes less.

Again, the growth of the lens is normally continuous throughout the whole period of life, as Priestley Smith has pointed out. This increased size of the lens probably accounts for some part of the acquired hypermetropia, for it is easily seen that if the lens be enlarged symmetrically, its focal length will also be increased.

A much more marked change is found to occur in the position of the punctum proximum : as age advances it gradually recedes, so that the amplitude of accommodation becomes less, as is indicated below. The diagram that is

given in most of the books professes to be founded on Donders' observations, but now, owing to the careful and laborious work of Dr. Alexander Duane and Dr. J. B. Thomas, his conclusions have been shown to be incorrect. Donders assumed emmetropia to be present without applying cycloplegic tests, and the number of cases he examined (fewer than 200) was too small for universally valid deductions to be made from them. Duane's results,

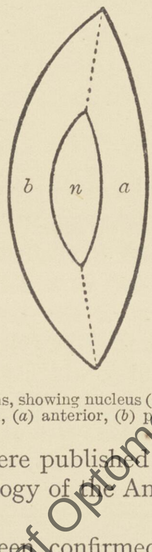

Fig. 1.—Diagram of human lens, showing nucleus (*n*) enclosed by the two diverging menisci, (*a*) anterior, (*b*) posterior.

based on 600 cases, were published in a paper read in the Section of Ophthalmology of the American Medical Association, June 1909.

The results have been confirmed by several thousand additional cases examined since. The diagram (*Fig.* 2) is a graphic representation of the values that he published in a paper on "Subnormal Accommodation" in the *Archives of Ophthalmology*, 1925, the thick line denoting the mean, the spaced line the maximum, and the dotted line the minimum values of A in dioptres. The near point is measured from a point 14 mm. in front of the cornea.

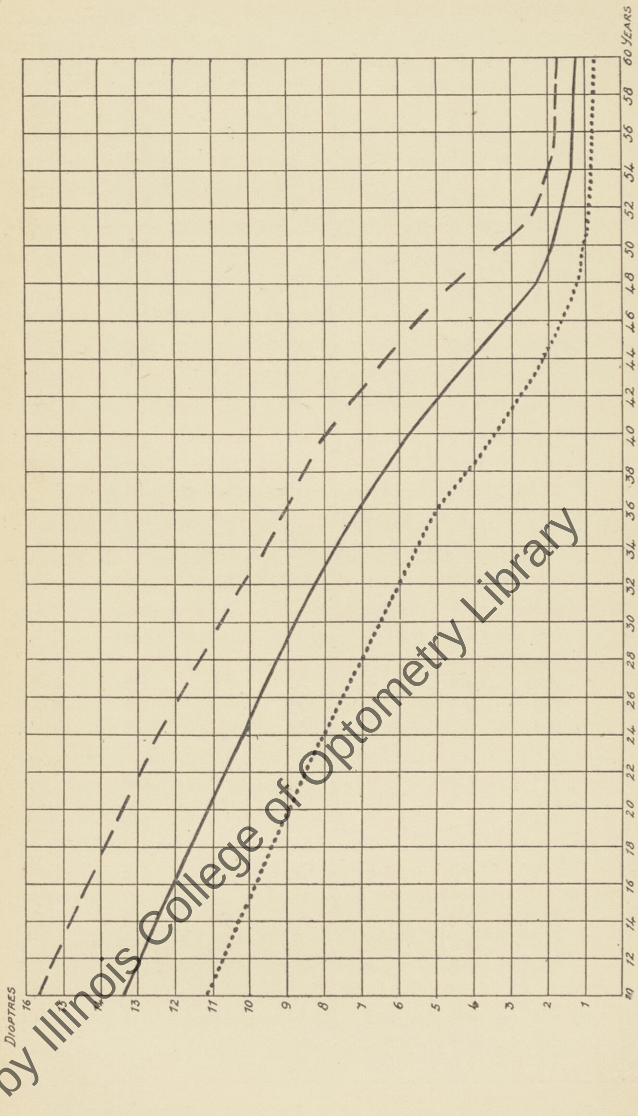

Fig. 2.—Graph showing amplitude of accommodation in dioptres according to age.

Amplitude of Accommodation in Dioptres.

Age	Minimum	Mean	Maximum	Age	Minimum	Mean	Maximum
				37	4·5	**6·7**	8·8
8	11·6	**13·8**	16·1	38	4·1	**6·4**	8·5
9	11·4	**13·6**	15·9	39	3·7	**6·1**	8·2
10	11·1	**13·4**	15·7	40	3·4	**5·8**	7·9
11	10·9	**13·2**	15·5	41	3·0	**5·4**	7·5
12	10·7	**12·9**	15·2	42	2·7	**5·0**	7·1
13	10·5	**12·7**	15·0	43	2·3	**4·5**	6·7
14	10·3	**12·5**	14·8	44	2·1	**4·0**	6·3
15	10·1	**12·3**	14·5	45	1·9	**3·6**	5·9
16	9·8	**12·0**	14·3	46	1·7	**3·1**	5·5
17	9·6	**11·8**	14·1	47	1·4	**2·7**	5·0
18	9·4	**11·6**	13·9	48	1·2	**2·3**	4·5
19	9·2	**11·4**	13·6	49	1·1	**2·1**	4·0
20	8·9	**11·1**	13·4	50	1·0	**1·9**	3·2
21	8·7	**10·9**	13·1	51	0·9	**1·7**	2·6
22	8·5	**10·7**	12·9	52	0·9	**1·6**	2·2
23	8·3	**10·5**	12·6	53	0·9	**1·5**	2·1
24	8·0	**10·2**	12·4	54	0·8	**1·4**	2·0
25	7·8	**9·9**	12·2	55	0·8	**1·3**	1·9
26	7·5	**9·7**	11·9	56	0·8	**1·3**	1·8
27	7·2	**9·5**	11·6	57	0·8	**1·3**	1·8
28	7·0	**9·2**	11·3	58	0·7	**1·3**	1·8
29	6·8	**9·0**	11·0	59	0·7	**1·2**	1·7
30	6·5	**8·7**	10·8	60	0·7	**1·2**	1·7
31	6·2	**8·4**	10·5	61	0·6	**1·2**	1·7
32	6·0	**8·1**	10·2	62	0·6	**1·2**	1·6
33	5·8	**7·9**	9·8	63	0·6	**1·1**	1·6
34	5·5	**7·6**	9·5	64	0·6	**1·1**	1·6
35	5·2	**7·3**	9·3	to		to	
36	4·9	**7·0**	9·0	72		**1·0**	

From childhood until the age of 40 the fall is nearly uniform, being ·2 D to ·3 D every year; from 40 to 48 the fall is more abrupt, the yearly decrement being ·45 D. After this age the descent becomes more gradual, so that after the age of 55 the amplitude of accommodation becomes practically stationary.

The mean values above are derived from a consideration of all the cases examined. It is clear that the mean was found to be nearer the minimum than the maximum value.

This loss of accommodative power is no doubt due to the increasing rigidity of the lens with age, so that the same contraction of the ciliary muscle no longer produces the same convexity of its surface.

Both these conditions, acquired hypermetropia and defective accommodation, are often included under the term *presbyopia*. Donders, however, restricts this term to "the condition in which, as the result of years, the range of accommodation is diminished and the vision of near objects is interfered with," and he regards the commencement of presbyopia as the time at which the punctum proximum recedes to 8 Paris inches (i.e., $8\frac{3}{4}$ English inches) from the eye. Much difficulty and confusion have arisen from the use of this word presbyopia; for instance, in one excellent book the presbyopia at 70 is given as + 5·5 D, and at 80 as + 7 D. The reader might therefore give an old man of 80, who could see well at a distance, + 7 D spectacles for reading. With these the unfortunate old gentleman would only be able to read a paper that was 6 in. from his eyes, and besides this inconvenience, his internal recti would cause symptoms of great fatigue from the excessive convergence that would be necessary to direct both eyes to so close an object.

It will save endless confusion if we agree not to use the word presbyopia, and if we consider the distance at which the patient is from his work. For playing the piano, for instance, the eyes should be about 20 in. ($\frac{1}{2}$ metre) away from the music; for reading, about 13 in. ($\frac{1}{3}$ metre) is taken as the ordinary distance; I find a distance of 30 cm.

or 11·8 in. from the spectacle plane to be more usual. Short people with short arms will find it more convenient to read at a distance of 10 in. ($\frac{1}{4}$ metre).

It is very important, then, to find the *working distance* at which the patient is going to hold his work. Further, we must find out his amplitude of accommodation, either by trial, or from his age and a reference to the table (*p.* 10). Now, although a patient may have 3 D of accommodation, it does not follow that he can use all his accommodation continuously; a man may be able to lift a 56 lb. weight, but it would be exceedingly irksome to carry such a weight for several hours at a time. In practice it will be found, as Landolt has pointed out, that $\frac{2}{3}$ of the accommodation (A′) can be used continuously by most people without fatigue (keeping $\frac{1}{3}$ in abeyance), provided that the amplitude of their accommodation is normal for their age. On this basis I have drawn up the following table for the working glass required at different ages for different distances from the plane of the spectacles:—

Age	Available Accommodation (A′)	Working Distance from the Spectacle Plane			
		20 in. ($\frac{1}{2}$ m.)	13 in. ($\frac{1}{3}$ m.)	11·8 in. $\left(\frac{1}{3\cdot33}\text{ m.}\right)$	10 in. ($\frac{1}{4}$ m.)
40	3·87	0	0	0	0·13 D
45	2·40	0	0·60 D	0·93 D	1·60 D
50	1·27	0·73 D	1·73 D	2·06 D	2·73 D
55	0·87	1·13 D	2·13 D	2·46 D	3·13 D
60	0·80	1·20 D	2·20 D	2·53 D	3·20 D

If the working distance be 13 in., a man of 60 will rarely require more than + 2·25 D in addition to his distance

glasses. Why should not + 3 D be given him to relieve his ciliary muscle entirely? It will be shown later (*p.* 126 *et seq.*) that it is far less fatiguing to converge both eyes to a point at a distance of $\frac{1}{3}$ m. when some effort of accommodation is made, than when the ciliary muscle is completely relaxed. In cases in which there is a tendency to excessive convergence it will be advisable to give a higher power than + 2·25 D.

It must be understood that the working glass must be added to that which fully corrects the patient for distance. As lenses are usually only made in strengths varying by a quarter of a dioptre, one would select the nearest equivalent glass. For instance, a myope of — ·5 D, aged 60, who holds his work 30 cm. from his glasses, will require — ·5 D + 2·53 D = + 2 D practically.

In every case one must find the highest convex (or the lowest concave) glass with which the patient can see distinctly the test types at 6 metres, and then the working glass is added to this to give the power of his reading glasses. In our clinical practice we pay no attention to that term of presbyopia which is called "acquired hypermetropia."

The table is very simply constructed. At the age of 50, A = 1·9D, of which $\frac{2}{3}$, or $\frac{2}{3}$ of 1·9 D, i.e., 1·27D are available for continuous use. For working at $\frac{1}{3}$ metre he will require + 3 D of adjustment, of which he can use + 1·27 D of his own accommodation; therefore he will require a glass of + 3 D — 1·27 D = + 1·73 D. Again, at the age of 60, A = 1·2 D, of which $\frac{2}{3}$, or ·8 D, are available; therefore for reading at 30 cm. or $\frac{1}{3{\cdot}33}$ metre's distance, he will require + 3·33 D — ·8 D = + 2·53 D.

Measurement of A.—When we have to measure the amplitude of accommodation (A) in a case, we must find the distance of the *punctum remotum* (R), and the distance of the *punctum proximum* (P) for each eye. When the vision, even after correction by the requisite glasses, is

very defective, it is clear that no accurate measurement of A can be made. The determination of R is called the determination of the static refraction, and will form the subject of the next chapter. This is the important problem that we always have to solve before prescribing any spectacles.

The determination of P may be effected in several ways, of which I will mention only two :—

1. The most usual plan is to give the patient his correction for distance and let him hold in his hand the small test type, and then note the shortest distance at which he can distinctly see the type with each eye separately. It will be found that he can only maintain his greatest accommodation for a few moments, so that the test need occupy but a minute or so.

The patient, when wearing his correction, may be regarded as an emmetrope(see *p*. 187) ; his $R = \infty$,

$$\therefore \frac{1}{R} = 0, \text{ so } A = \frac{1}{P} - \frac{1}{R} = \frac{1}{P} - 0,$$

and we have only to determine P, or rather $\frac{1}{P}$

Now if the amplitude (A) be great, or P be small, a small variation in P will make a very great variation in A. For instance 3·28 in. is $\frac{1}{12}$ m., but 3·58 in. is $\frac{1}{11}$ m., so in such a case an error in measurement of $\frac{3}{10}$ in. would entail an error of 1 D in the value of A.

All difficulty is avoided by adding a concave glass of power G to the distance correction. An example will make this quite plain.

Suppose that we are examining a young hypermetrope whose distance correction has been found, and when this is worn, his near point appears to be somewhere in the neighbourhood of 3 or 4 inches ; the patient will get tired, and time will be wasted if any attempt be made to measure

P accurately. At once diminish the distance correction by, say -6 D, ($G = -6$), if P then be found to be 20 cm. or $\frac{1}{5}$ m., the true value of $\frac{1}{P}$ without the additional concave glass will be

$$A \text{ or } \frac{1}{P} = 5 - G = 5 - (-6) = 11.$$

This result may be confirmed by substituting -5 D for the -6 D lens; if P is then found to be $16\frac{2}{3}$ cm., or $\frac{1}{6}$ m., the true value of $\frac{1}{P}$ without this G will be

$$A \text{ or } \frac{1}{P} = 6 - (-G) = 6 - (-5) = 11.$$

By this simple device we have made a variation of $3\frac{1}{3}$ cm. instead of 0·762 cm. ($\frac{3}{10}$ in.) correspond to a variation of 1 D.

I personally use for this test my modification of Landolt's ophthalmo-dynamometer. It consists of a square black plate with a narrow vertical slit in the middle. On one side is Duane's standard test-object, a small white card 3 mm. × 1·25 mm. which is bisected by an engraved black line 3 mm. long by ·2 mm. thick; on the reverse side is a similar object, only double the size, for those whose visual acuteness is diminished. This is supported in a holder to the handle of which a steel tape-measure is attached. The tape is marked on one side in centimetres and on the other with the reciprocals of the metric lengths, i.e., opposite 10 cm. or $\frac{1}{10}$ metre is the number 10; opposite 20 cm. is the number 5; opposite 50 cm. is the number 2, and so on. On holding the steel tape against the spectacle frame that carries the requisite distance correction, and allowing the patient gradually to withdraw the instrument from the eye that is being examined until the engraved line becomes distinct, P can be read off in centimetres, and on the reverse side we have $\frac{1}{P}$ or the dioptric value of the

refraction exercised. It is essential that one eye should be examined at a time, the other being closed. If we wish to determine the distance of the punctum proximum from the eye, we must measure the distance of the frame from its first principal point, which is about 1·3 mm. behind the cornea; more frequently, however, we have to determine the power of the equivalent lens when placed in its usual position (about 13·6 mm. in front of the cornea); in that case the distance must be measured from this position. There is no definite position for the spectacle plane. In Germany they place it 12 mm. in front of the cornea, in England it is usually said to be 15 mm. before the cornea, although practically it is often placed as near the cornea as the eyelashes will allow. I have taken 13·6 mm. as the standard distance, as this will avoid contact with the eyelashes, and 12 mm. is certainly too short a distance for our eyelashes.

This little instrument will also be found to be very useful in determining the range of convergence (p. 121).

2. *Scheiner's Method.*—This depends on the phenomena that are observed when a diaphragm with two pinholes is placed before the eye. The pinholes must be close together, not further apart than about $\frac{1}{8}$ in. (3 or 4 mm.), and one of the holes should be covered with a small piece of coloured gelatin. We will suppose the colour chosen to be red. The patient holds the diaphragm close to his eye, so that the pupil is immediately behind the two pinholes, and he is directed to look at a point of light, say the light reflected from a thermometer bulb. If his near point be 12 inches from his eye, and the point of light be at this distance, he will see a single point of light. If, however, his eye be nearer the point of light, he will see two points of light, one white and one red. The nearer the eye is to the object, the wider will be the separation of the images. When the red hole is placed on the right side of the pupil, the red image will appear to be on the left

side of the white image, and *vice versa.* This is what is called *crossed* or *heteronymous diplopia.* The reason of this phenomenon will be evident from the adjoining diagram (*Fig.* 3). It will be seen that with a myopic patient if the point of light be beyond his far point, the red image will appear on the right side of the white one, in other words the diplopia will be *homonymous.*

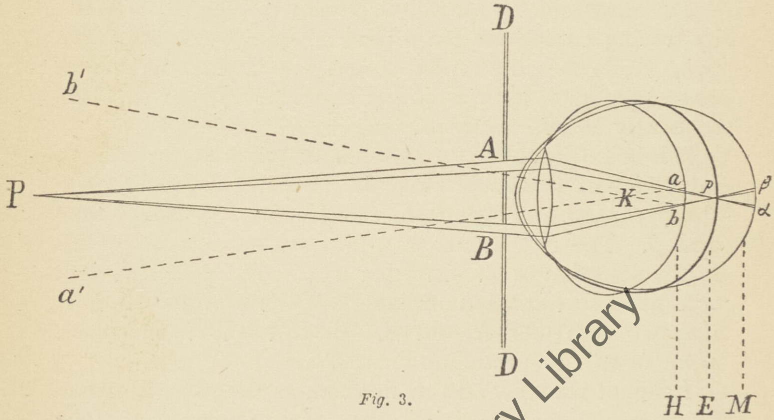

Fig. 3.

In the diagram DD represents the diaphragm with its two pinholes A and B; behind it is the eye with retina, at H if hypermetropic, at E if emmetropic, and at M if myopic. P is a point of light at the punctum proximum of an emmetropic eye. Diverging rays from P will be seen to reach the diaphragm D, and through the two holes at A and B two pencils will pass, being converged by the refracting system of the eye towards the focus p on the retina of the emmetropic eye. Therefore only one point will be seen by the emmetropic eye, when P is at its punctum proximum.

If P be within the distance of the punctum proximum of the eye, the rays from P will be so divergent that the eye will tend to bring them to a focus behind the retina.

In other words, H will represent the retina in this case, and the two converging pencils will form images on the retina at *a* and *b*. They will really be confusion circles, but as the pencils transmitted through the holes are very thin, the images will be fairly distinct. These images will be projected outwards in the reverse direction, as are all retinal images.

The direction in which the object is seen is obtained by tracing a line from the retinal image through the nodal point K, so that the retinal images at *a* and *b* will be seen as two points of light at a' and b'. The diplopia will consequently be *heteronymous.*

If P be further from the eye than its punctum proximum, for which it is accommodated, the pencils will cross in the eye at *p* and will form small confusion circles at α and β. The retinal image α will be seen as an object in the direction of b'. In other words, two points of light will be observed, seen in the positions corresponding to the two apertures A and B. In this way *homonymous diplopia* will be produced.

I cannot recommend this method, as the patient cannot be trusted always to hold the two pinholes exactly in front of his pupil, and only one image will be seen, whatever the distance, when only one pinhole is opposite his pupil.

Insufficiency of Accommodation.—Under this head are comprised all cases in which the value of A is below the mean value of A for their age. A man of 42 with a manifest A of 4 or 4·5 D may be able to read with ease without special reading glasses, but one of 20 with only a manifest A of 6 D or so can hardly read at all.

There are three types :—

1. *Faulty Ciliary Muscle.*—This is almost always found in myopia unless of very low degree, and usually gives rise to no symptoms, except that special reading glasses will be required at an earlier age than usual, if glasses are required at all for close work.

2. *Static Insufficiency.*—This is due to a lack of response of the lens to a normal innervation of the ciliary muscle. The lenses of such patients are older than their years ; a graph of their accommodation, say from 38 onwards, will resemble very closely the normal graph from 42 or so onwards. The characteristics of this condition are that : (1) There are no sudden variations from week to week ; and (2) Both eyes are affected, although the insufficiency may show itself in one eye before the other ; in that case each eye must be given its appropriate reading addition.

There are no symptoms which are not immediately removed by giving the necessary glasses.

3. *Dynamic Insufficiency.*—This is due to defective innervation : (*a*) With absent or defective pupillary reaction to light ; (*b*) Without loss of the light reflex.

a. The dilatation of the pupil and loss of accommodation due to cycloplegic drugs, such as atropine, are too well known to discuss here, and their action is purely local on the nerve-endings or on the muscle itself. Sometimes one eye alone is affected, as from a unilateral ophthalmoplegia interna, which is usually ascribed to a nuclear lesion, but few cases have been pathologically examined. It is clear that the condition might arise from lesions in the short ciliary nerves, the ciliary ganglion, or its short motor root without any implication of the trunk of the third nerve. I am not aware of any examination having been made of the ciliary ganglion in such cases.

Accommodative insufficiency occurs frequently in connection with reflex iridoplegia (Argyll Robertson pupil). The great majority of all these cases are syphilitic in origin.

b. Dynamic insufficiency without loss of the light reflex is fairly common ; it is, however, very frequently associated with convergence insufficiency, and hence sometimes with an inverse Argyll Robertson pupil—i.e., the pupil does react to light, but not when attempting to view an object close at hand. There is also in some cases a

transient contraction of the field of view. Frequently there is a marked conjunctival injection and engorgement of the larger retinal vessels ; the former adds a great deal to the patient's discomfort. It is usually bilateral, though occasionally one eye is more affected than the other. It is probably due to a lesion or functional disorder of the higher centres or supranuclear tracts.

It very frequently occurs in encephalitis lethargica and diphtheria, and has been observed in botulism ; in such cases its sudden onset in both eyes at the same time and the absence of iridoplegia usually distinguish this form of insufficiency from those of syphilitic or tuberculous origin.

The following conditions have been noted as possible causes : eyestrain, exposure to intense light, prolonged microscope work, or working in a poor light ; nasal obstruction, sinus disease, and oral infection. Neurotic subjects are especially prone to this affection. Insufficiency of accommodation will be often found during convalescence from any debilitating illness, such as influenza, but in its most marked form it appears as a sequela of diphtheria. It frequently occurs after what has been diagnosed as follicular tonsillitis, especially in hospital practice ; whether this has been due to a mild diphtheritic infection or not I have no idea, but I am inclined to think its origin is diphtheritic if there is any paralysis of the soft palate, rendered evident by the nasal character of the speech. Finally, I have found it not infrequently in young people, schoolboys, who are addicted to self-abuse.

The *symptoms* of accommodative insufficiency, besides the blurring of vision, are headache, pain in the eyes with smarting and burning of the eyelids, and a difficulty in concentrating attention on what is read. The conjunctival trouble is apt to be extremely obstinate unless appropriate correcting glasses are given.

Objects often appear smaller than usual (micropsia), as the increased effort to exercise accommodation would lead

the patient to think that the object is nearer than it really is, were it not that the retinal image is not increased. Hence objects appear smaller.

This, however, is not a complete explanation, as patients often say that objects seem to be small and very far away. Now objects of known size, if they appear small are presumably at a great distance ; if they appear near, there will be a considerable amount of convergence exercised. In this case, as an unusual effort of accommodation is made while no excessive convergence is called into play, there is a confusion of mental judgement as to whether the objects are small or at a great distance. The final inference usually made is that the objects are both smaller and further away than they really are.

Treatment.—Correction of the refractive errors is far the most important item, with especial attention to any difference there may be in the accommodative power in the two eyes. In functional cases exercise of the accommodation and the convergence have an excellent effect. Cover the left eye and look with the right at a needle held in the hand at arm's length ; now bring the needle steadily nearer until it begins to blur. Repeat the same process with the other eye ; and then with both eyes until it doubles. "Each exercise consists of this triple approximation repeated three or four times and the patient does two or three such exercises a day." Strychnine internally or by hypodermic injection is often of use, and I personally think that pilocarpine instillations twice a day hasten recovery. The prognosis in cases of diphtheritic paralysis and in those after sore throat is always good ; indeed, it usually passes away in one or two months without any treatment.

The greater part of this section is due to Duane's excellent paper on "Subnormal Accommodation" in the *Archives of Ophthalmology*, 1925, to which I have already alluded.

CHAPTER II

STATIC REFRACTION

THE refraction of the eye when the accommodation is relaxed is called the static refraction.

If when the accommodation is completely relaxed the refraction of the eye is such that parallel rays come to a focus on the retina, the condition is called *emmetropia*. In other words, an emmetrope can see distinctly distant objects without exerting his accommodation, provided that no pathological lesion or functional incapacity exists. If parallel rays come to a focus behind the retina, the condition is *hypermetropia*, and if in front of the retina *myopia*.

If the static refraction is not emmetropic the condition is called *ametropia*.

Axial hypermetropia is due to the axial length of the eyeball being too short, *axial myopia* is due to its being too long; both of these conditions are included under the term *axial ametropia*, and they are by far the commonest immediate cause of hypermetropia and myopia.

Alterations of the curvature of the cornea are sometimes found: a flattening of the cornea causing *curvature hypermetropia*, while *curvature myopia* is due to too pronounced a curvature. Astigmatism is caused almost always by a different amount of curvature of the cornea in different meridians, though its sudden onset may be due to the lens of the eye being tilted when dislocated; astigmatism will be dealt with later on.

Sometimes an alteration of the refractive index of the media causes an error of refraction. This "index ametropia," as it is called, is not of much practical importance, and receives but scant treatment in the books. Indeed, little

or no work has been done as yet in determining these refractive indices in cases of disease with such changes.

Sometimes in diabetes a sudden change in the static refraction of the eye occurs, often associated with astigmatism, myopia with a sudden increase (hypermetropia with a decrease) of blood-sugar. These changes appear to be due to a deformation of the lens from osmotic processes and to index changes in the cortex of the lens. (Duke-Elder, *Brit. Jour. Ophthalmol.*, ix, p. 167).

The so-called "second sight", which sometimes occurs before the advent of cataract, is probably due to a change of the index of the lens. It is well to point out here that *hypermetropia* would result from :—

1. Lowering the index of the aqueous alone. For clearly the posterior concave surface of the cornea would have its diverging power increased, although, at the same time, the anterior surface of the lens would gain in converging power. It must be remembered that the power of the cornea is more than $2\frac{1}{2}$ times greater than that of the lens, so the net result is that the converging power of the eye is reduced.

2. Raising the index of the vitreous alone. For the posterior power of the lens would be reduced.

3. Raising the index of both the aqueous and the vitreous (even though the index of the aqueous may be slightly greater than that of the vitreous).

4. Raising the index of the cortex of the lens, or lowering the index of the nucleus. (*See Fig.* 1, *p.* 8.)

Myopia results under reversed conditions.

To this class belongs what Demicheri has described as false lenticonus, which is the only index anomaly that has been definitely established up to the present. In his case when the pupil was dilated, the central part was found to be very myopic (— 10 D or more), while the peripheral zone was hypermetropic (+ 3 D or so). The surfaces of the lens and its nucleus showed no irregular curvature, as they formed quite regular reflection images. The anomaly

is explained by a great diminution of the index of the cortex, and hence an increase of the central power of the lens.

Two other spurious forms of hypermetropia may be mentioned: aphakia, the condition of the eye when the lens has been removed; and the suddenly acquired hypermetropia, due to detached retina, which, however, does not demand any treatment with glasses.

Tests of refraction may be divided into subjective and objective. Before applying any accurate subjective test it is most necessary to exclude all pathological errors except those due to errors of refraction. For this purpose nothing is better than the pinhole test, of which too little use is now made. We will, however, begin the description of the subjective tests with an account of an old-fashioned qualitative test that I personally have never found useful, although it is worth describing from its intrinsic interest.

SUBJECTIVE TESTS OF AMETROPIA.

Chromatic Test.—This depends upon the fact that the eye is not truly achromatic, red light coming to a focus behind that of blue light. Ordinarily this is of no inconvenience, but if a purple glass that only transmits red light and blue light is used, the chromatic aberration of the eye may be made evident. The patient is directed to look at a point of light through such a glass with one eye, the other eye being closed. In cases of hypermetropia the point of light will appear blue surrounded with a red ring, but when myopia is present the light will appear reddish and surrounded with a blue ring.

Pinhole Test.—A black diaphragm having a small perforation in its centre will be found in all boxes of trial lenses, but the perforation is usually too small, it should be about $1\frac{1}{2}$ mm. wide. One eye of the patient is covered, and he is then asked to read from the assigned distance (usually 6 metres) that line of the test types which is the smallest that he can see. If he cannot read the line that

he should, he is given the pinhole diaphragm, and told to hold it close to his eye, and to look at the test types again through the hole, while the other eye remains covered.

If his sight is improved, that eye quite certainly suffers from some error of refraction ; if the sight is not improved, the failure of vision will be due to some other defect, and we may suspect that the transparency of the media or the retinal sensibility is defective.

The reason is obvious. The pinhole only gives passage to a very narrow pencil of rays, and therefore the size of the confusion circles on the retina must be smaller. It is true that less light will enter the eye, and therefore the brightness will be less ; but the image, and consequently the vision in ametropia, will be much more distinct. If the diameter of the aperture be $\frac{1}{5}$ that of the pupil, the diameter of the circle of confusion on the retina will be $\frac{1}{5}$ that of the usual circle of confusion. Suppose, for instance, that owing to a refractive error, the rays diverging from a point of light come to a focus behind the retina, so that on the retina the converging pencil covers an area occupied by 100 cones at the macula ; with a pinhole $\frac{1}{5}$ of the diameter of the pupil before it, the area covered will be $\frac{1}{25}$ smaller, and only 4 cones will be covered ; consequently the acuteness of vision will be enormously increased.

Further, a test of the kind of refractive defect, and a rough estimate of its amount, may be made with this useful little device, although I do not recommend this plan for accurate results.

When the pinhole is held say 3 in. in front of an ametropic eye, and the patient moves it slightly up or slightly down, while his attention is directed to the distant test types (the other eye being of course closed), he will notice that the test types appear to move. If the motion is in the same direction as that of the pinhole, the vertical meridian of his eye will be myopic ; if the motion is in the opposite direction, that meridian will be hypermetropic.

Similarly, by moving the pinhole from side to side, the refraction of the horizontal meridian may be tested. If the motion in one direction is more extensive than in the other, astigmatism is present, the greatest error occurring in that meridian in which the motion is most rapid and extensive. Should the apparent motion of the test types be *with* the motion of the pinhole from above downwards, and *against* the motion from side to side, mixed astigmatism is present, a concave glass being required from above downwards and a convex glass from side to side. The presence of astigmatism of 1 D or so can be easily detected in this way.

On putting up before the eye (between it and the pinhole) the correcting glass, the test types will appear to be stationary, however the pinhole is moved. I do not think the test can be relied upon to give results with less error than 1 D in any but the most intelligent patients.

The explanation of the phenomena is obvious from a consideration of the diagram (*Fig.* 3).

The aperture is first held in the position A, and in a hypermetropic patient the image of the distant point tends to be formed at a; when the hole is depressed to B the image is at b. These retinal images are projected externally at a' and b'. Hence in hypermetropia, on moving the hole downwards, the object viewed appears to move upwards from a' to b'. If the eye be emmetropic, the retinal image is formed at p, wherever the aperture is; consequently the object appears stationary. In myopia the retinal image at α will be seen as an object near b', and that at β as an object near a', as explained above. Therefore, in myopia, the object will appear to move *with* the pinhole, and in hypermetropia *against* the pinhole.

Visual Acuteness.—The acuteness of vision is a function of the nervous mechanism of the eye and brain; the acuteness may be good although the refractive error

be great ; in such a case, on correcting the refractive error with glasses the vision will be of standard amount. The first point to decide is the answer to the question, What is *standard* visual acuteness ?

This is a far more difficult question than it appears at first sight. As we are dealing only with refractive errors that may be corrected by glasses, all defects of light sense and of colour sense are here excluded from consideration ; further, no examination of the peripheral parts of the field is made, for it will be found that the accurate correction of the macular errors of refraction scarcely improves peripheral vision at all. Hence for our present purpose the term is usually understood to mean the *resolving power of the macula* under ordinary illumination.

It is agreed by all competent observers that a double star cannot be recognized as such by any eye if the visual angle between the two stars be less than 30″, and that it requires good eyesight, and absence of glare from a neighbouring star, to separate the two stars even if the visual angle between them be 60″. On this ground the standard *minimum separabile* is taken to be 1′.

But it has been found that two black lines cannot be separated, i.e., cannot be distinguished as such, unless the angle of separation amounts to 90″ or 100″. On the other hand, Professor E. M. Barnard, of the Yerkes Observatory, a most reliable and accurate observer, found that he could detect a long wire at such a distance that its diameter only subtended an angle of 0·44″. Here, of course, it was not his *form* sense that was being tested, it was only evidence of his extraordinary sensibility to *light difference*. This is a point that is continually obscuring the conclusions to be drawn from tests of the *minimum visibile*. A black line one twenty-thousandth part of an inch wide, on a white surface, can be recognized by a good eye at a distance of 10 inches, which would lead one to infer that the *minimum visibile* was an angle of 1″ ; but here again it is a contrast effect, or a light difference test. The *aligning* power of the eye, which is called upon in the use of verniers, slide-rules, and rangefinders, is found to show an accuracy in some cases to an angle of 1″, and in all normal cases to an angle of 3″.

In order that two objects may be seen apart it is usually said that their macular images must be separated by at least one unexcited macular cone. Now the diameter of a macular cone subtends at K (*Fig.* 3), the posterior nodal point of the eye, an angle of about 24″. The *minimum separabile* should theoretically be twice this angle, or about 48″. But as blur circles occur round every retinal image, and it is very doubtful if the eye can be kept absolutely fixed upon any point, the *minimum separabile* is presumably greater than this angle ; so, as already stated, the standard angle accepted by ophthalmologists is 1′.*

The test types recommended by the Committee of British Ophthalmologists are of oblong shape (5 by 4) ; some letters have been excluded as being too difficult or too easy to decipher, and the selected letters are printed without serifs. Each letter of the line that should be read at 6 metres' distance subtends an angle of 5′, while the distinguishing characteristic of the letter subtends an angle of not less than 1′ at that distance. For instance, the central mark of the letter E subtends an angle of 1′. The line above consists of larger letters which subtend the same angle 5′ at a distance of 9 metres. Above this are lines of increasingly larger letters that a standard eye should read at distances of 12 m., 18 m., 24 m., 36 m., and 60 m.

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* It might be thought that the resolving power of the eye should be dealt with in the same way as that of a telescope or a microscope, by taking account of the diffraction rings. It will be found that this method gives a *lower* limit than that indicated in the text, which depends upon the size of a macular cone. For instance, the diffraction formula for the minimum angle is $\theta = 1{\cdot}22\,\frac{\lambda}{a}$ or $\left(\frac{5}{a}\right)''$ where a is the aperture in inches, which is Dawes' rule for the resolving power of telescopes when light in the middle of the spectrum is considered. If the pupil be $\frac{1}{4}$ in. in diameter, of course the minimum visual angle would be one of 20″. This refers to an emmetropic eye, and does not invalidate the statement made on *p.* 25 which deals with the improved definition that results from the use of a pinhole with an *ametropic* eye.

A patient who at 6 metres' distance can only see the line which he should read at 12 metres is said to have a *vision* of $\frac{6}{12}$. If, on providing him with a correcting glass, he can read the 6-metre line, his *visual acuteness* is said to be $\frac{6}{6}$, or standard. It is assumed that rays proceeding from a point at a distance of 6 metres, or 20 feet, may be regarded as parallel ; they are so nearly parallel that the defect is quite negligible, so that a patient who has standard vision at 6 metres has also standard vision at an infinite distance.

The illumination of the test types must not be less than 3 *foot candles*, although a higher illumination up to 10 *foot candles* is recommended to allow for deterioration of the lamps, etc.

It is true that Druault, who uses different units, found that with 1·5 *metre candles* illumination the $\frac{15}{15}$ line could just be read, and with 0·25 *metre candle* the visual acuity fell to $\frac{15}{30}$; yet it required an increase of illumination to 16·7 *metre candles* to raise the visual acuity to $\frac{15}{12}$; so that no error will be incurred by increasing the illumination up to the limit of 10 *foot candles*.

As some letters are much easier to discern than others, Landolt has provided us with some more accurate test types, formed of broken rings resembling the letter C; the gap subtending an angle of 1′ at the assigned distance of 5 metres. The sizes of the figures are so arranged that, at 5 metres' distance, each of them corresponds to a different acuteness of vision: ·1, ·15, ·2, ·3, ·4, ·5, ·6, ·7, ·8, ·9, 1, 1·25, 1·5, 1·75, 2 (*Fig.* 4).

One objection to Landolt's test is this: that those who have a very good light sense will be able to see the gap, or at any rate notice an increased illumination at its situation, in a broken ring that denotes a higher visual acuity than they really have. Another objection is that so much time is wasted in explaining to patients how to indicate the position of the gap, that its use is practically debarred in hospitals or when one is pressed for time.

When the patient cannot see the 60 m. line of Snellen's types at the prescribed distance of 6 m., he is allowed to approach the types until he can see the largest letter.

If this occur at 3 m. we enter his uncorrected vision as $\frac{3}{60}$. When Landolt's types are used his vision would be noted as $\frac{3}{50}$, not as ·06, which does not record the distance 3 m.

The system of radiating lines in the diagram (*Fig.* 5) is a most useful test for astigmatism, and is hence called the astigmatic fan. The method of using it is explained on *p.* 39 ; meanwhile it will be sufficient to say that if all

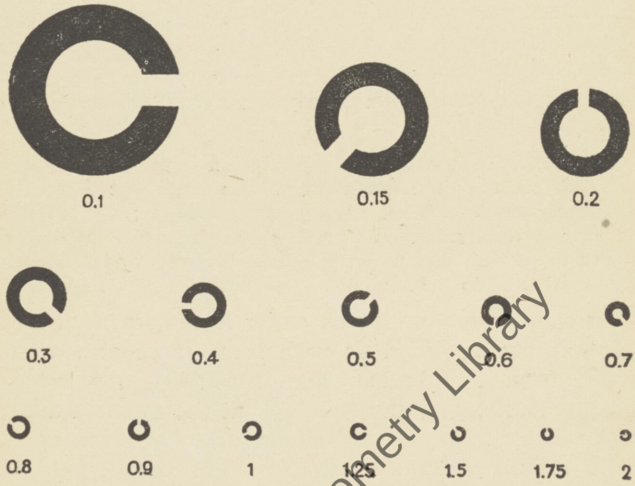

Fig. 4.—(In this figure the sizes of Landolt's test types have been reduced, so that the distance at which they should be viewed is 5 feet.)

these radiating lines appear *equally* distinct, astigmatism is probably not present.

We must carefully distinguish between the terms *vision* and *visual acuteness*. The first refers to uncorrected vision, while the second refers to the vision after correction. V = 1 should be termed *standard*, not *normal* vision, for it is by no means the average vision ; misapprehensions often arise in the law courts from the use of the term *normal*.

As we know that convex glasses magnify and concave glasses diminish the apparent size of objects, it would naturally be thought that the retinal images of a corrected hypermetrope would be larger than those of a corrected myope. It will be proved later (*p.* 167) that if the ametropia be axial this is not the case when the correcting glasses are placed in the anterior focal plane of the eye. Under these conditions the retinal images are of precisely the same size as those of an emmetrope without glasses. Unfortunately spectacles are not worn exactly

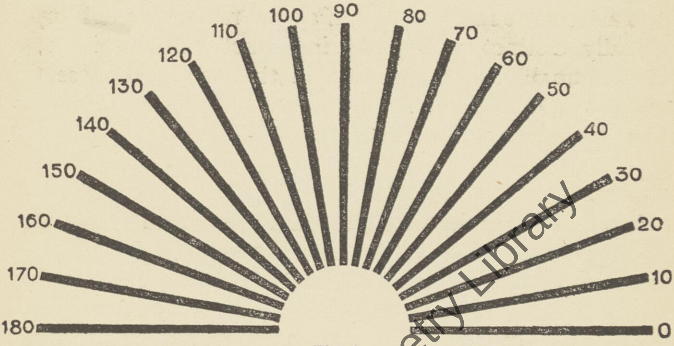

Fig. 5.

in the anterior focal plane of the eye, but usually about 2 mm. behind it. The anterior focal plane of the eye is 15·7 mm. in front of the cornea. I find that most people prefer to wear their glasses about 13·6 mm. from their eyes. However, if it is required to make punctiliously exact determinations of the visual acuteness, the correcting glasses can be set in the stated position (15·7 mm.) and then the results of tests for visual acuteness in all cases of axial ametropia are strictly comparable.

Subjective Test of Visual Acuteness.—By the pinhole test we have found that there is some refractive error to correct We therefore place the patient at 6 metres'

distance from the test types, which should be well illuminated, and placing the trial frame on his face we cover up his left eye with a diaphragm so as to exclude it. We then ask him to read the smallest type that he can see.

1. If he can read $\frac{6}{6}$ he must be either emmetropic or hypermetropic (or possibly very slightly astigmatic).

Holding up + ·5 D or + 1 D before his eye, if we find that he can still read $\frac{6}{6}$ he is certainly hypermetropic, and we go on trying stronger and stronger convex glasses until the 6 m. line is blurred. We *then* direct his attention to the astigmatic fan, and note if all the radiating lines appear equally distinct, or equally indistinct. If this is so, he is presumably not astigmatic in that eye. Suppose that with + 1·75 D the 6 m. line is just blurred, we note down that his manifest hypermetropia is + 1·5 D

or $$\text{R.H}^{m} = +1{\cdot}5\,\text{D}$$

when + 1·5 D is the strongest glass with which he can see $\frac{6}{6}$.

This is not, however, the total hypermetropia, for if the accommodation be paralysed by the instillation of gutt. atrop. sulph. 1 per cent three times a day for two or three days, a much stronger glass will be required to enable the patient to read $\frac{6}{6}$. We might find that + 4·5 D were required. It is clear that the total hypermetropia is + 4·5 D, of which + 3 D is *latent*. The hypermetropia of the patient has always been corrected by the accommodation, so that although the necessity for this is removed by offering him glasses, there remains a tonic contraction of the ciliary muscle which renders + 3 D latent. In such cases the best treatment is to prescribe lenses which correct all the manifest and one-third of the latent hypermetropia. It is true that such glasses may need changing for stronger ones in two or three years, as the tonic contraction of the ciliary muscle relaxes, but the glasses will be comfortable to wear from the first.

It will be found that the younger the patient the more of the total hypermetropia is masked or latent. In some cases the whole of the hypermetropia is latent, or even an apparent myopia may be induced by an excessive contraction of the ciliary muscle. Such a condition is often called *spasm* of accommodation, and will probably require a prolonged use of atropine instillations for treatment. A good indication of the presence of much latent hypermetropia is afforded by finding the position of the near point (*punctum proximum*).

For instance, suppose a young girl of 15 presents herself with rather small pupils, and reddened eyelids. She complains that her eyelids feel heavy and that her eyes often ache and water. She has a nearly constant headache, which is worse in the evening after doing needlework. She is able to read $\frac{6}{6}$ with each eye separately, but she can also read the same line with $+ 4$ D. To determine the position of her near point, the glasses are taken off, and we find that she cannot read small type nearer than $8\frac{1}{2}$ in. from her eyes, say 8 in. or 20 cm. from the spectacle plane.

$$\frac{1}{P} = \frac{100}{20} \text{ m.} = 5, \text{ and if Hyp.} = 4 \text{ D}, \frac{1}{R} = -4$$

so $A = 5 - (-4) = 9$ D

Now at 15, A is about 12·3 D (*see p.* 10); she then must either have a subnormal accommodation or a much higher degree of hypermetropia than $+ 4$ D. It is most probable that a great part of her hypermetropia is latent. We therefore instil atropine and defer her examination until another day. In this actual case I found on paralysis of her accommodation that her total hypermetropia was $+ 7{\cdot}5$ D. I ordered her to wear $+ 5$ D constantly, as that corrects all her manifest ($H^m = + 4$ D) and practically $\frac{1}{3}$ of her latent H,

$$(H^l = H^t - H^m = + 7{\cdot}5 \text{ D} - 4 \text{ D} = + 3{\cdot}5 \text{ D})$$

i.e., $4 \text{ D} + \frac{1}{3}$ of $3{\cdot}5 \text{ D} = + 5$ D approximately.

3

As $\frac{1}{R}$ is now found to be $-7{\cdot}5$

$$A = 5 - (-7{\cdot}5) = 12{\cdot}5\ D$$

The note made in my notebook was :—

R & L V $\frac{6}{6}$

$H^m + 4\ D$, $H^t + 7{\cdot}5\ D$, $A = 12{\cdot}5\ D$

Ord. $+ 5\ D$

The Fogging System.—It is a good plan, when testing a patient, to give him an over-correction at first. In hypermetropia, convex lenses of such strength are first put in the trial frames that the vision is reduced to, say, $\frac{6}{18}$. By slowly reducing the strength of these lenses, satisfactory vision of $\frac{6}{6}$ may be at last obtained with higher powers than would have been attained by gradually increasing the strength of the glasses. By this method we often succeed in getting a greater relaxation of the ciliary muscle, and so a higher amount of hypermetropia is made manifest.

After finding the highest convex glass with which the patient has the best vision with each eye separately, we should always try the vision with both eyes together, and see if a still higher glass will be tolerated.

For example, $\frac{6}{18}$ is read with each eye separately when $+ 4\ D$ lenses are put in the trial frames ; on gradually reducing the strength to $+ 3\ D$ each eye can then read $\frac{6}{6}$. When both eyes are used together it is found that $\frac{6}{6}$ can still be read when $+ {\cdot}5\ D$ glasses are held before the trial frames.

The entry in the notebook should take the form

$$\left.\begin{array}{l} R\ H^m + 3\ D\ \frac{6}{6} \\ L\ H^m + 3\ D\ \frac{6}{6} \end{array}\right\}\ H^m + 3{\cdot}5\ D\ \frac{6}{6}$$

If no higher amount of hypermetropia is found by the other tests, and when there is no evidence of astigmatism, these glasses may be prescribed.

Astigmatism can only be accurately determined by objective methods, but the most reliable subjective method

for suggesting the presence of small amounts of astigmatism is by means of the *crossed cylinder*. This should always be used if subjective methods of examination are employed alone.

The crossed cylinder in its most usual form consists of a convex cylinder $+ \cdot 25$ D on one surface, and on the other surface a cylinder of $- \cdot 25$ D with its axis set at right angles to the axis of the convex cylinder. It consequently has the effect of a $- \cdot 25$ D sph. combined with a $+ \cdot 5$ cylinder; and by placing it before the final correction for each eye ($+ 3$ D in the above case) with one axis first vertical, then at 45°, and rotating it slowly through an angle of 180°, one can find if the vision is improved in any one of these positions. If there is no improvement of visual acuteness in any position, and still more certainly if there is deterioration of vision, one may reasonably conclude that no evidence of astigmatism can be obtained by subjective methods.

Crossed cylinders are also made with cylindrical surfaces of ·13 D, and ·5 D; for myself I rarely use the form with surfaces of ·13 D.

2. Suppose that our patient's vision is below the "standard," and that he can only read the fourth line of the test types ($\frac{6}{18}$), and that this line is blurred with a weak convex glass. He must then be either myopic, astigmatic, or he may have spasm of accommodation.

The last condition, a high degree of latent H, is revealed by the position of the near point, by the instillation of atropine, and by the objective tests to be presently described.

If a concave glass improves his vision, and if the near point is closer to his eye than it should be at his age, we may assume him to be myopic, but he may also be astigmatic. Astigmatism, as before, must be first excluded by the use of the fan.

For example, a patient of 35 who is not astigmatic, sees

with each eye $\frac{6}{18}$ and with -2 D he can see with each eye separately $\frac{6}{6}$. We find that his near point is about 5 in., or $\frac{1}{8}$ metre, distant. Now at 35 he should have 7·3 D of accommodation, therefore we might expect him to have -1 D of myopia, for $8-1=7$. In myopia the amplitude of accommodation is sometimes less than the normal amount, owing to the want of exercise of the ciliary muscle. On trying him with concave glasses we find that with -2 D he can see $\frac{6}{6}$ with each eye, and that with $-1{\cdot}5$ D before both eyes together he can still see $\frac{6}{6}$. We therefore enter in our note book

$$\left.\begin{array}{l} \text{R V } \frac{6}{18}\,\text{M}^{\text{m}} - 2\,\text{D}\ \frac{6}{6} \\ \text{L V } \frac{6}{18}\,\text{M}^{\text{m}} - 2\,\text{D}\ \frac{6}{6} \end{array}\right\} - 1{\cdot}5\,\text{D}\ \frac{6}{6}$$

and prescribe $-1{\cdot}5$ D for his use.

In all cases of young myopes it is important to paralyse the accommodation before prescribing glasses, as otherwise one is almost certain to over-correct the myopia. To allow for the tone of the ciliary muscle one should add $-{\cdot}5$ D to the glasses which give the best vision under atropine.

3. Should no spherical glasses materially improve the sight, when the pinhole test has indicated an error of refraction, the patient is certainly astigmatic.

ASTIGMATISM.

This may be either regular or irregular.

Irregular Astigmatism is usually due to the irregularities left on the cornea after ulcers and nebulæ, but it may be caused by conical cornea or some other defect. The diagnosis of any marked case is readily made in the following way. The patient, facing the window, is made to look in such a direction that the surgeon sees its reflected image on that part of the cornea that is in front of the pupil. If the image is perfect, the surface of the cornea at this part is normal and regular. If the pupillary

area of the cornea is irregular or abnormally curved, the reflected image of the window is distorted in shape. The vision will be exceedingly defective, and the examination of such cases is most tedious and unsatisfactory. Little or no help is given by objective tests, and great patience is required to discover the most suitable glasses. I think the best result can be obtained by the use of the stenopaic slit, which consists of a black diaphragm with a slit about 2 mm. wide in the middle line. Placing it first in the horizontal direction in the trial frames immediately before the pupil (the other eye being covered), we find out how much of the test types the patient can see, and the strongest convex (or the weakest concave) glass that gives him the best vision with the slit horizontal. We then repeat this procedure with the slit in the vertical direction, and again in oblique directions at angles of 45° and 135°. Great care must be taken that the slit in the vertical or oblique positions is exactly in front of the centre of the pupil. If the patient can see through the slit without tilting his head, it may be assumed that it is in the right position.

Suppose that, when the slit is horizontal, his best vision is attained with + 1 D. We then try, by altering the position of the slit a little one way or the other to see if his vision is improved. If we find that the exactly horizontal position is the best, we may regard + 1 D as giving the best correction in the horizontal meridian. Now, on rotating the slit to the vertical direction, we try again with various glasses. Suppose we find that — 4 D gives the best vision; we can then infer that the best glass we can order will be one that is convex to the extent of + 1 D in the horizontal direction, and concave to the extent of — 4 D in the vertical direction, i.e., a + 1 D sph. — 5 D cyl. plane axis horizontal.

However, on trying this glass, it is more than likely that the patient will not see so well as with a simple spherical lens and the slit. In such a case we can, if he

prefer it, order a spherical lens with one of its surfaces blackened except for a central linear opening. Of course, with such a slit, the field of view is enormously diminished, but the patient may prefer better sight with diminished field. Patient trial is the only method by which even approximately satisfactory results can be obtained in irregular astigmatism.

Regular Astigmatism.—This occurs when the refraction of the eye is a maximum and a minimum in two meridians which are at right angles to each other. It is usually due to the cornea having a different curvature in these two meridians. The cornea is never truly spherical even in emmetropia, but it may be regarded as resembling the top that one cuts off the breakfast egg. Now if one were to cut off, instead of the top, a piece from the side of the egg, it would resemble an astigmatic cornea. It would be more curved from side to side than from above downwards. Clearly, if the greatest curvature of the cornea be from above downwards, the greatest refraction would be in the vertical meridian, and the least in the horizontal meridian. This is called astigmatism "with the rule," and is the most common form. It is corrected by a convex cylindrical glass, the plane axis of which is in the emmetropic meridian.

The greatest curvature may be in an oblique or a horizontal meridian. This last condition is called astigmatism "against the rule," and commonly gives rise to more pronounced symptoms of asthenopia than the usual form.

A similar astigmatism of the lens sometimes occurs from an unequal contraction of the ciliary muscle; it is always, I believe, in the reverse direction to that of the cornea, and appears to be Nature's attempt to correct the corneal astigmatism.

If a patient be astigmatic "with the rule," his horizontal meridian being emmetropic and his vertical meridian

myopic, on looking at the letter T he will notice that the vertical limb of the letter is distinct, but the horizontal limb is blurred. The explanation of this phenomenon requires a little thought. A line is regarded as distinct when its sides are well defined, while the definition of its ends is of little importance. Therefore, on looking at a vertical line, all that is necessary to see it distinctly is to get good definition of its width, i.e., to have the horizontal meridian of one's eye properly focused. If the vertical meridian of our patient's eye be myopic, it is clear that he will not be able to define clearly the width of a transverse line ; he will see its terminations clearly, but he will be unable to define its upper and lower margins.

On this principle is based the subjective test for astigmatism. The patient is directed to look at a system of lines radiating from a centre in directions varying from 0° to 180° (*Fig.* 5). The " fogging " system is the best to adopt. The highest convex (or the weakest concave) glass is given that just enables him to distinguish one set of lines. The strength of this glass is reduced until he can see one of the lines distinctly, while the rest are indistinct, especially those at right angles to the distinct one. The distinct line gives the meridian of *greatest* curvature. Let us suppose it is the line at 80° from the horizontal. On now holding in front of the trial frames concave cylinders, with their plane axes at right angles to the 80° line (i.e., at an angle of 170°), of gradually increasing strength, we shall eventually find that cylinder with which the 170° line can be defined. This is the correction.

On putting the cylinder at this angle into the trial frames we try the effect of a weak convex or a weak concave glass (or the crossed cylinder) held in front of the combination, and make any slight alteration with which he can read more of the test types. If he see better with his head tilted to the right, he will require the cylinder slightly rotated to the right, and *vice versa*.

Astigmatism may be also tested with the stenopaic slit as described on *p.* 37, but it should be remembered that no test for astigmatism is reliable without the instillation of a mydriatic, and it is well not to waste too much time on subjective tests, as a more accurate determination can be made in a far shorter time by the objective tests.

NOTATION OF CYLINDRICAL LENSES.

This used to be a very vexed question, but fortunately now there is an international agreement : (1) The meridians of astigmatism are measured and represented as the observer looks at the patient ; (2) The radius vector is supposed to rotate counter-clockwise from the horizontal position. This notation is easily understood from the adjoining diagram (*Fig.* 6, *see also Fig.* 5, *p.* 31). Angles of 30° and 100° mean angles measured counter-clockwise from the initial line OX. Trial frames which have the angles marked on the semicircular arc below the glasses must therefore be graduated counter-clockwise, starting from 0° on the observer's left.

This method is universally used by all mathematicians for positive angles, and by all manufacturing opticians. Further, it avoids all source of error when using the astigmatic fan or the astigmometer, and removes all difficulties in printing reports of cases. When bilateral symmetry is present, the sum of the angles is always 180° (e.g., R 80° and L 100°), so no trouble arises from this point.

Transposition of Cylinders.—If an eye be hypermetropic in the vertical meridian to the extent of + 2 D, and to the extent of + 4 D from side to side, we can correct the error by either + 2 D sph. + 2 D cyl. ax. 90°, or by + 4 D sph. − 2 D cyl. ax. 0°. Clearly, the former glass would be

the easier to make, and would be the lighter. The rule for transposing such a prescription is the following :—

The new spherical power will be the sum of the old spherical and cylindrical powers, while the new cylinder will have the same power as the old cylinder, but of opposite

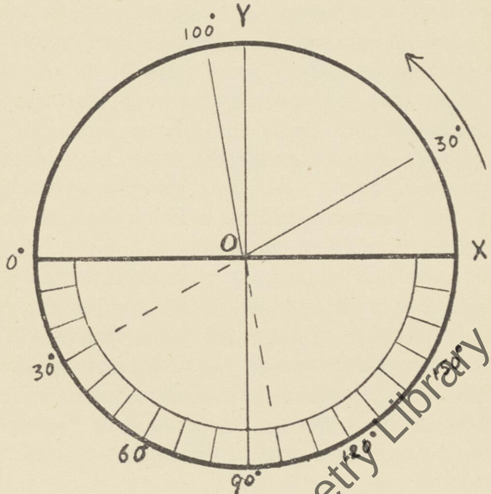

Fig. 6.

sign, and its axis will be at right angles to that of the old cylinder.

Thus + ·5 D sph. — 3 D cyl. ax. 15° is equivalent to — 2·5 D sph. + 3 D cyl. ax. 105°.

Now, although the last form would be heavier and would entail more grinding, its periscopic effect would be much greater if worn with the concave spherical surface next the eye. (*See pp.* 71–4.)

OBJECTIVE TESTS OF AMETROPIA.

There are several ready tests that we may apply with the ophthalmoscope to determine the nature of the ametropia. We will first describe these before dealing with the methods for precisely determining its amount.

1. *The Concave Mirror at a Distance.*

a. Let us suppose that the mirror is held about a metre from the patient in a dark room, and if, on throwing the reflected light into his eye, one can see the disc or some of the vessels without using the convex lens, one may be sure that there is a considerable error of refraction. If *myopia* greater than -1 D be present, a real inverted image of the patient's fundus will be formed at his far point, and as this then happens to be on his side of the mirror, one will see a part of it at any rate.

In *emmetropia* no image will be formed, as the emergent rays will be parallel, and at any distance one will only be able to receive one beam of parallel rays, corresponding to a single point of his illuminated retina.

In *hypermetropia* a virtual image will be formed behind the patient's head, and part of this erect magnified image will be seen.

If the observer now move his head (and the mirror) from side to side, the vessels that he sees will move in the same direction in cases of hypermetropia, when the image is erect, but they will move in the opposite direction in cases of myopia, when the image is inverted (*p.* 218).

b. On slightly turning the mirror, the pupillary red reflex is seen to move *with* the mirror in myopia > -1 D, but against the mirror in hypermetropia. (*See* RETINOSCOPY, *pp.* 46, 47.)

2. *The Indirect Method.*—Let the ophthalmoscope be now brought about 18 in. from the patient, and on holding the convex lens close to the eye, an inverted image of his fundus will be seen in the usual way. Unless

the observer be myopic, he will probably see the image by the indirect method more easily if he put up a + 3 D lens behind his ophthalmoscope.

If he now slowly withdraw the lens from the patient's eye, he will notice that the image appears to increase in size in myopia, but to diminish in hypermetropia. The greater the change in size, the greater is the ametropia. If, when the disc is viewed, as the lens is slowly withdrawn from the eye, the image of the disc become more oval, astigmatism is present. The diameter that increases the most indicates the meridian which is most myopic ; that which diminishes the most is the most hypermetropic. If no change be noticed in the size of the disc on moving the lens, the eye must be nearly emmetropic.

When the convex lens is close to the eye the image is greater in hypermetropia than in myopia.

The explanation of these points is given in the optical section of this book (*pp.* 215–221).

Amount of Ametropia : Direct Method.—There are three conditions which must be fulfilled before this can be regarded as a satisfactory method. The accommodation both (1) of the patient, and (2) of the observer, must be relaxed, and (3) the ophthalmoscope must be held about ½ in. from the patient's cornea, so that the correcting glasses that are required may be placed in the first focal plane of the patient's eye. The observer must first correct any existing ametropia in his own eye by an appropriate glass.

If the patient be *emmetropic*, the light from his eye will emerge in parallel beams, and if the previous conditions are observed, a distinct image of his fundus will be seen, which will be blurred by the addition of the weakest convex glass.

If the patient be *myopic*, the fundus will appear indistinct until the lowest necessary concave glass has been added.

Similarly, in *hypermetropia* the highest convex glass must be put up that will enable the observer to see the fundus distinctly.

In *astigmatism* a different glass will be required to get a distinct view of vessels running in different directions. Thus if + 3 D be the strongest glass with which the *horizontal* vessels are seen distinctly, + 3 D is the refraction in the *vertical* meridian. If the vertical vessels can be defined with + 5 D, this indicates the refraction in the horizontal meridian. This is much the readiest method of estimating a patient's refraction, but it requires a good deal of practice to be at all proficient at it. Even experienced ophthalmologists cannot attain the same accuracy in their results as can be reached by the next method described.

It is important to remember that, though the optic disc is the easiest object to examine, it is the very part to avoid in an estimate of a patient's refraction, as it corresponds to his blind spot. The macular region is the part of which the refractive error should be determined; but this is the very part in which there are no pronounced details to observe. It is true there are usually some horizontal vessels going towards the macula from the disc, but at the macula itself there are no vessels to be seen. Finally, in estimating astigmatism, it is most unlikely that the patient should happen to have two vessels near the macula at right angles to each other that lie exactly in the meridians of his astigmatism. I cannot therefore recommend this method, although it should be practised and checked by retinoscopy, as the ability once acquired is invaluable in estimating the amount of swelling present in a case of optic neuritis, or the forward displacement of a detached retina.

Should the observer be ametropic, he must deduct his own error of refraction from the correcting glass used behind the ophthalmoscope to determine the patient's

ametropia. Thus if the observer be myopic to the extent of $-$ 2 D, and find that he sees R fundus with $-$ 1 D, L with + ·5 D,

$$\text{R ametropia is } -1\,\text{D} - (-2\,\text{D}) = +1\,\text{D}.$$
$$\text{L ametropia is } +\cdot5\,\text{D} - (-2\,\text{D}) = +2{\cdot}5\,\text{D}.$$

Retinoscopy.—This is by far the most accurate and perfect way of objectively testing a patient's refraction. I shall therefore describe the method in full detail, as there are several minutiæ about its correct performance that do not seem to be generally known. The principle of the method is simple enough, as will be seen from the adjoining diagram (*Fig.* 7).

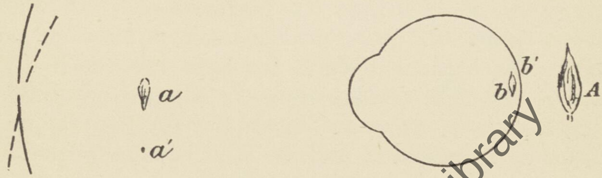

Fig. 7.

Most oculists advocate the use of the plane mirror; after carefully testing my results when using both the plane and the concave mirror, I have found that they are considerably more accurate when I use the concave mirror. The illumination is better and the troublesome swirling shadows described later (*p.* 49) are less annoying than when the plane mirror is used. I shall therefore give a detailed description of retinoscopy with a concave mirror of focal length 25 cm. For further remarks on the plane mirror, *see p.* 54.

The light *A* (*Fig.* 7) should be a small brilliant electric lamp, or an ordinary lamp screened by a diaphragm with a circular hole, about 8 or 10 mm. wide, opposite its most

brilliant part. With this exception the room should be dark, with dark-coloured walls. I have been told that dark brown is a much more suitable colour than black, though unfortunately I have not as yet had any experience of it. The light *A* should be placed behind and above the patient's head, say 125 cm. or 150 cm. from the observer's concave mirror; a real inverted image of the light will then be formed at *a*, 31 cm. or 30 cm. from the mirror, which will illumine a small area of the patient's retina at *b*; in fact an inverted image of *a* will tend to be formed in the neighbourhood of *b*. The exact position of *a* is of no importance, but it is essential for accurate work that the concave mirror should be 1 metre from the patient's spectacle plane. The observer, seated on a stool with his eye close behind the perforation of the mirror, which is held at the prescribed distance, will see the pupil a red colour when he reflects the light towards it. This is due to the light coming from the illuminated patch of the fundus at *b*; of course he cannot see the real image at *a*, for he is behind it, and the light is passing from *a* to the patient, so this causes no confusion.

Now, on turning the mirror slightly downwards, the image at *a* will move downwards to *a′*. Consequently the illumined area on the patient's retina will move upwards to *b′* in every case, whether the patient be myopic or hypermetropic. If the patient's distance from the observer be less than that of his far point, the observer will see a magnified erect image of the patch of light through the patient's pupil. We will suppose that the observer is seated at the distance of 1 metre from the patient, so that, by leaning back a foot, the distance between his and the patient's eye will be 133 cm., and by leaning forwards 8 in. the distance will be reduced to 80 cm. It is clear, then, that if the mirror is held at 1 metre's distance from the patient's eye, and if, on turning the mirror, the light is seen to move in the reverse direction

across the pupil, that eye must be hypermetropic, emmetropic, or myopic to a less extent than 1 D. If the myopia be greater than 1 D, an inverted image of *b* will be formed at the patient's far point, which is situated somewhere between him and the observer. This inverted image of *b* is what the observer will see apparently in the pupillary area, and as it moves in the reverse direction to *b*, it will move in the same direction as the mirror.

Suppose a patient, whose eyes are fully dilated with atropine, keep his eyes fixed on the mirror, and, on turning the mirror from side to side, the red reflex is seen to move across the pupil in the same direction as the mirror; concave glasses are placed in the trial frames before the patient's eyes, until one is found that just reverses the movement of the light. It will be found that as the right correction is approached, the red reflex becomes brighter and the movements more rapid, so that, after a little practice, one can judge from the first glance at an eye, by the appearance of the reflex and of its movements, whether the ametropia is high or not, i.e., whether to begin by putting in, say, a 6 D or a 2 D lens. In this way much time may be saved. We will suppose in the given case that, from the appearance of the reflex, we decide on trying − 3 D first. The reflex looks much brighter and moves more quickly. On substituting a − 3·5 D lens, we find on leaning back that the light moves rapidly in the same direction as the mirror, but on leaning forwards the light moves against the mirror. We know, therefore, that the patient's far point is now more than 80 cm. and less than 133 cm. from his eye. On holding the mirror at 1 metre's distance, we find that the pupil is fully illuminated, but on slightly turning it the reflex suddenly disappears without our being able to say in which direction the light has moved. We may be sure that when this occurs the mirror is exactly at the far point of the eye, i.e., 1 metre, and therefore we say that, on adding − 1 D to the glass

in the frames, that meridian of the eye will be corrected for distance, or infinity as it is often called.

When the mirror is exactly at the far point of the eye, a definite image of the illuminated patch at *b* is formed in the plane of the mirror; but the observer will see the brilliant light only when it happens to fall over the small perforation in the mirror, a very slight rotation of the mirror will throw the image on the mirror beyond the hole, when it will no longer be visible. This explains the brilliant illumination and its sudden disappearance on slightly tilting the mirror.

It will be found that, by adopting this procedure, we can in most cases accurately determine with an error of less than ·25 D the refraction of an eye. For instance, in this case if we add + ·25 D to the — 3·5 D in the frames, we shall find that the light moves with the mirror at a distance of 133 cm. and also at that of 1 metre, but that at a distance of 80 cm. the movement of the light becomes indeterminate, showing that now — 1·25 D must be added to the glasses in the trial frames in order to correct the horizontal meridian of the eye for infinity ($80 = \frac{100}{1{\cdot}25}$). If on turning the mirror from above downwards, exactly the same observations are made, we conclude that no astigmatism is present. In practice it is often more convenient for changing the lenses in the patient's trial frame to start the retinoscopy at a distance of only 80 cm. until one gets very near the "point of reversal," and then take the correct distance of 1 metre for an exact result. It should be noted that for accurate work one must tilt the mirror very slowly when near the reversal point, and watch for the movement of the weak shadow that follows the light as it passes across the pupil. The observer should, if necessary, correct his refraction for the distance of 1 metre by putting up the appropriate lens behind his ophthalmoscope mirror. We can then confirm our result by trying the patient with the test types when provided with — 4·5 D.

As $133{\cdot}3 = \frac{100}{{\cdot}75}$ it will be seen that, by varying the

distance of the mirror from 133 cm. to 80 cm., one really has a range between ·75 D and 1·25 D ; and, according to the position in which the indeterminate movement of the light occurs, one knows whether to add — ·75 D, — 1 D, or — 1·25 D.

In some cases the light seems to move "all ways." The central, prepupillary, part of the cornea is nearly always more spherical than the periphery, and hence the movement of the light in the peripheral region may be opposed to that in the central part. It is always necessary when this difficulty occurs with a dilated pupil to fix one's attention solely on the movements of the light in the central part. The introduction into the spectacle frame of a diaphragm with a central perforation 5 mm. in diameter is said to get over this difficulty, but I have found it quite useless ; one cannot see whether the patient is looking directly at the mirror or elsewhere, and of course it is the refractive error of the central part of the widely dilated pupillary area one must correct.

In conical cornea the illuminated part of the pupil appears to be surrounded with peculiar swirling shadows ; when there is irregular astigmatism and spherical aberration, two shadows may appear on opposite sides of the pupil, and on tilting the mirror they move towards each other. This is called "the scissors movement" ; it may be sometimes obviated by getting the patient to look in a slightly different direction, or by slightly inclining the mirror. In any case in which these anomalous shadows are troublesome, no such precisely accurate determination of the refraction can be made.

Aberration.—We know that if parallel rays fall on an ordinary spherical lens, the peripheral rays intersect at a point that is nearer the lens than the focus of the more central rays. The same thing usually occurs with the eye. Dr. Tscherning has devised a simple little instrument for subjectively testing the aberration of the eye. His "aberroscope" consists of a planoconvex lens on which is etched a micrometer scale in the form of little squares. Priestley

Smith's " keratometer " serves the purpose admirably. A distant light is viewed through the instrument when it is 4 or 5 inches away from the eye; if the peripheral lines appear concave outwards (" pincushion distortion "), aberration is present; if the peripheral lines appear convex outwards (" barrel distortion "), there is an over-correction. Sometimes it will be found that the pupil shows an aberration in one part and an over-correction in another part.

Regular Astigmatism.—Suppose that when — 3 D is put in the trial frames we see a bright band of light running across the pupil at an angle of about 10° or 15°, we may be sure that astigmatism is present and that this band denotes one of the axes of astigmatism. On replacing the — 3 D by a — 3·5 D lens, and turning the mirror in the direction of this band (at an angle of 10° or 15°), suppose we find that the light moves with the mirror on leaning back, but against the mirror on leaning forwards, we discover as before that — 3·5 D must give the reversal point and that — 4·5 D must be the correcting glass for this meridian.

On now turning the mirror at right angles to the band, we see a very distinct movement of the light with the mirror, i.e., in the meridian at an angle of 100° or 105°. The most usual and the easiest method to adopt is now to add concave spherical glasses until this meridian is corrected. I have, however, found an enormous advantage in stopping short of the full correction of this second meridian with spherical lenses; when near the reversal point I substitute for the spherical lens a cylindrical lens of the same power with its axis placed in the meridian I guess to be appropriate. It will be seen that by this means I can find the correct axis with extraordinary accuracy in all cases in which an exact correction can be given. Suppose that on placing a — 1·5 D sph. in front of the — 3·5 D sph. in the trial frame I find that when tilting the mirror downwards in the 100° meridian the light band moves much more quickly in the same direction, I know

that I am near the reversal point in this meridian. Of course if I sway the mirror from side to side, the light moves in the opposite direction to that of my mirror, but with that I have no concern now, as I have found that — 3·5 D gives the reversal point for this lateral meridian. I at once replace the — 1·5 D sph. by a — 1·5 D cyl. with its plane axis at 10° in the trial frame. I now *slowly* tilt the mirror downwards in a meridian at right angles to the plane axis of the cylinder. This is easy to do, as one can see the ground glass border of the cylinder that indicates the line of the axis. If the light bounded by a shadow moves downwards but not in the same line as that in which I move my mirror, I know that the axis of my cylinder is wrong, and the power is also too weak. Now it is essential to correct the axis first. Suppose that the movement of the light is obliquely downwards in the 120° meridian; this would suggest to the novice that the axis of the cylinder should be 30° instead of 10°, but this will be found erroneous, as the procedure greatly magnifies the error. It is well to regard about a quarter of the suggested error as approximately correct. The suggested error is 20°. I try moving the cylinder 5°, so that its axis now lies at 15°. If I now find that the light moves down at right angles to the axis of the cylinder, while I move the mirror down in that meridian, I know that the axis of my cylinder is correctly placed, but it is still insufficient in power. If I find that — 2 D cyl. placed with its axis at 15° gives the point of reversal at the distance of 1 metre for this meridian, I shall see that now the whole pupil will be fully illuminated, and on turning the mirror in any direction the light will suddenly disappear; in other words, the point of reversal is obtained at the distance of 1 metre by — 3·5 D sph. — 2 D cyl. ax. 15°. The correction under homatropine for distance will therefore be

— 4·5 D sph. — 2 D cyl. ax. 15°.

"If my correction of astigmatism is *wrong in amount* (e.g., if — 2·25 D cyl. instead of — 2 D cyl. is required), as I approach the patient from the distance of 133 cm. I shall, at a certain distance from him, find that in the meridian of 100° the light moves with the mirror, while in the meridian of 10° it moves against the mirror. When this happens, I simply change the strength of the cylinder until reversal takes place at just the same distance from the eye for all meridians alike.

"Finally, if my *spherical lens* is at fault, then reversal takes place, evenly indeed in all meridians, but either when I am too close to the eye or too far from it. I then alter the strength of the spherical accordingly, until reversal takes place at just one metre."

The last two quoted paragraphs I have practically taken from Duane's paper on "The Systematic Use of Cylinders on Making the Shadow Test" (*The Ophthalmic Record*, 1903).

The optical principles on which my method of determining the axis of astigmatism is based will be explained in the optical section of this book (*pp.* 193-5). I may here point out that if the trial cylinder used be of power $\frac{7}{8}$ of the power really required, while the error of its axis be 5° as before, the suggested error is six times greater than the true error. For instance, if in the case of an astigmatism of — 2 D cyl. ax. 15°, my retinoscopy had been done with — 1·75 D cyl. ax. 10°, the light would have sheared off in the 130° meridian, i.e., the suggested error would have been 30° or *six* times the real error of 5°. If a cylinder of the true power be put up in the wrong axis, it is exceedingly difficult to adjust it successfully by retinoscopy, as the magnification of the error is greater still and as one gets nearer the true axis the movement of the light is more difficult to follow. The following is a short table in which the fractions of the suggested error given are roughly correct.

Partial Correction of any Cylinder	Error 5°	Error 10°	
$\frac{7}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	Fractional part of suggested error
$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	

I must warn the reader that he will meet with some cases in which some irregular astigmatism occurs, when these precise methods will not work. It may be some satisfaction to him, if not to his patient, to know that then no exact optical correction can be given.

There are a few further points that should be mentioned when dealing with retinoscopy.

1. For all accurate work, the patient should look directly at the mirror; for it is his macula that we wish to correct, not some other less sensitive part of his retina. This is especially important in cases of high myopia, when there is frequently a posterior staphyloma at the posterior pole, so that the glass which corrects his optic disc may differ from that required to correct his macula by two or three dioptres. Hence, atropine or homatropine should always be used unless contra-indicated, as otherwise the pupil will strongly contract when the light reflected from the mirror is viewed. However, for the convenience of the patient, he may be allowed to look above one's head until the final correction is almost attained.

2. It will be seen from the diagram (*Fig.* 7, *p.* 45) that when the mirror is only a few inches away from the patient's eye, so that it is within its focal distance of 25 cm., the real image of the mirror will tend to be formed on the retina itself, and hence the light will move in the same direction as the mirror, even though the eye be highly hypermetropic.

On this principle is founded Lister's retinoscope, which consists of a concave mirror of focal length $1\frac{1}{2}$ to 2 metres. When used at a distance of 1 metre from the patent, the light moves *with* the mirror in hypermetropia, but *against* the mirror in myopia. It gives a better illumination than the plane mirror, and is recommended to those who habitually use the plane mirror.

3. When a plane instead of a concave mirror is used, all the movements are reversed, for only a virtual erect image is formed by it, and hence it will move up when the mirror is moved down; consequently, in hypermetropia the light moves with the mirror, and in myopia against the mirror.

The plane mirror reveals lenticular opacities and irregular astigmatism more readily than the concave mirror, but these are just the points one wishes to neglect when doing a retinoscopy. For myself, I prefer the concave mirror, as the troublesome swirling and cross shadows that are occasionally seen are less prominent with it than with the plane mirror. The use of the plane mirror is now almost universal, as it is said to give more accurate results, and special modifications, such as Lister's or Macnab's, are provided to render the light reflex more definite. I am convinced that the ordinary concave mirror will invariably give as accurate results as any provided that the observer is seated at the distance of 1 metre from the patient. Preference for one or the other form is to a great extent a matter of habit, and does not really justify any dogmatic assertion of its superiority.

The Ophthalmometer.—This is an instrument for measuring the corneal curvature in any meridian, and hence it is used to estimate the amount of regular corneal astigmatism present. Instruments of so many different patterns are now on the market that a general description of their principle must suffice. Two objects called 'mires' are so placed that their images, reflected by the cornea,

are viewed through a telescope, inside which is an arrangement for doubling the images seen. There are therefore four images visible, but the attention of the observer is to be fixed upon the two central images. First, the meridian of least corneal refraction is found, by rotating the revolving disc or arc on which the mires are fixed until that position is found which shows the central images of the mires at their greatest distance apart. By turning the adjusting screw the mires are then approximated until their images just touch. Both the mires are bisected by a black line, and the final adjustment is made by a slight rotation of the arc that carries them, until these two black lines are continuous as one black line that lies across their central images. The angle at which the arc is now set is read off from the pointer, say it is 10°, and a note is made of it, as this angle gives the axis of the *concave* correcting cylinder. Now when the arc is rotated slightly, if corneal astigmatism be present, the black line will appear to be broken and at the same time the corneal images will overlap. On rotating the arc 90° from its initial position (i.e., until the reading is 100°), the overlapping will in a case of regular astigmatism have reached its maximum, while at the same time the two black lines have again become continuous. By the extent of this overlapping the corneal astigmatism can be directly read off. In most instruments one of the mires is marked in steps, so that each step corresponds to one dioptre of astigmatism.

In the best instruments a scale is provided which enables one to read off in millimetres the radius of curvature of the cornea in the meridian observed. As a scientific instrument it is admirable for determining the curvature of the cornea, but unfortunately the astigmatism given by the instrument does *not* give the power of the cylinder required in the correcting spectacles. It merely gives the power of the cylinder which, if placed *in contact* with the cornea, would correct the anterior corneal astigmatism.

As I am assuming that the glasses will be worn at the distance of 13·6 mm. from the cornea, I have calculated the following table of the true cylinder required to correct the corneal astigmatism as indicated by the ophthalmometric reading. (For the method of calculation *see pp.* 181–4, where more complete tables are given.)

Convex Spherical Correction (D).

D	Ophthalmometric Reading							
	− 1	− 2	− 3	− 4	− 5	− 6	− 7	− 8
0	−1·014	−2·056	−3·128	−4·230	−5·365	−6·533	−7·737	−8·977
+ 2	− ·959	−1·944	−2·956	−3·997	−5·067	−6·168	−7·301	−8·467
+ 4	− ·906	−1·836	−2·790	−3·771	−4·778	−5·813	−6·878	−7·974
+ 6	− ·854	−1·730	−2·629	−3·551	−4·498	−5·471	−6·470	−7·497
+ 8	− ·804	−1·628	−2·473	−3·339	−4·227	−5·139	−6·075	−7·036
+10	− ·755	−1·529	−2·321	−3·133	−3·965	−4·819	−5·693	−6·592
+12	− ·708	−1·433	−2·175	−2·935	−3·712	−4·502	−5·326	−6·163
+14	− ·663	−1·340	−2·034	−2·743	−3·468	−4·211	−4·971	−5·750
+16	− ·619	−1·251	−1·897	−2·557	−3·233	−3·924	−4·630	−5·353

Concave Spherical Correction (D).

D	Ophthalmometric Reading							
	− 1	− 2	− 3	− 4	− 5	− 6	− 7	− 8
0	−1·014	−2·056	−3·128	−4·230	−5·365	−6·533	− 7·737	− 8·977
− 2	−1·070	−2·171	−3·304	−4·470	−5·672	−6·910	− 8·186	− 9·503
− 4	−1·128	−2·289	−3·485	−4·718	−5·988	−7·299	− 8·651	−10·047
− 6	−1·187	−2·411	−3·672	−4·972	−6·314	−7·699	− 9·129	−10·607
− 8	−1·248	−2·535	−3·863	−5·233	−6·647	−8·110	− 9·622	−11·185
−10	−1·311	−2·663	−4·060	−5·502	−6·993	−8·534	−10·129	−11·780
−12	−1·375	−2·794	−4·261	−5·777	−7·346	−8·970	−10·651	−12·393
−14	−1·440	−2·929	−4·468	−6·061	−7·709	−9·417	−11·187	−13·023
−16	−1·507	−3·067	−4·680	−6·351	−8·082	−9·877	−11·738	−13·671

It is seen that the ophthalmometer gives no indication of the amount of hypermetropia or myopia present. As the table is made out for *concave* cylinders it is necessary to know the highest convex (or the lowest concave) lens

that will correct the error for the meridian of the least corneal curvature. This is easily obtained by the 'fogging' system used in the subjective test of astigmatism (*p.* 39).

It will be found that hypermetropes, except in some cases of mixed astigmatism, will require a weaker concave cylinder than that indicated, while myopes will require a stronger cylinder. For instance, a cataract patient may have − 8 D astigmatism indicated by the instrument, but if it were found that his meridian of least corneal refraction required + 14 D to correct it, he would require a cylinder of only − 5·75 D; for that cylinder placed 13·6 mm. in front of the cornea in combination with a + 14 D sph. would have the same cylindrical effect as a − 8 D cylinder with its appropriate spherical lens when placed in contact with the cornea. Should, however, a myope of − 14 D in his least refractive meridian give the same indicated astigmatism of − 8 D with the instrument, it would be found that he would require a cylinder of − 13 D. It is clear that without a table of this sort the readings given by the ophthalmometer will be grossly erroneous if they are regarded as indicating the cylindrical correction required for glasses.

But even with such a table the total astigmatism of the eye is not given, the lenticular element is entirely neglected. It would, therefore, have failed to detect the astigmatism in Young's eye (*p.* 2). Further, no attention is paid to the astigmatism that may exist on the posterior (concave) surface of the cornea. This is exceedingly difficult to determine. Tscherning has done some valuable work on the subject, and has found, in the few cases he has examined, a certain amount (·5 D or so) of astigmatism, which has always been "against the rule"; this condition is very probably general.

Weiland gives the case of a myope who with − 7 D sph. had V = $\frac{6}{6}$ and refused any minus or plus cylinder, yet on two examinations with the ophthalmometer she "showed a corneal astigmatism

of -2 D with the rule and ought to have been corrected by a cylinder of $-2{\cdot}5$ D ". He quotes another case who had subjectively $-4{\cdot}5$ D sph. -1 D cyl. ax. 180° V = $\frac{20}{20}$, yet the ophthalmometer indicated -2 D cyl. *against the rule*. Such cases may not be very common, but certainly the subjective examination with trial lenses must never be neglected, as far more reliance should be placed on those glasses with which the patient sees best. There is still another objection to the instrument, for the corneal reflections occur at points about 1·25 mm. on either side of the centre of the cornea, so the curvature of the apex of the cornea is not given. However, it is a valuable method for the finer diagnosis of the construction of the eye. If the astigmatism of the posterior surface of the cornea be regarded as negligible, the difference between the actual astigmatism of the whole eye and the corrected reading of the ophthalmometer would give some idea of the amount and axis of the lenticular astigmatism. In chronic glaucoma there is usually a progressive hypermetropia, although as the lens is pushed forwards in this condition one would expect myopia to result. In the few cases that I have examined with the ophthalmometer I have found that the curvature of the cornea was steadily diminished to such an extent that, in spite of the myopia induced as indicated above, the resulting hypermetropia was fully explained by this alteration of the shape of the cornea owing to the tendency of the whole globe to become more spherical from the internal pressure.

I have dealt at some length with the inaccuracies of the ophthalmometer in estimating astigmatism. Its advantages may be summarized in this way :—

1. Economy of time. The examination by an adept should not take longer than one minute for each eye.

2. A mydriatic is unnecessary.

3. In aphakia it is extremely useful.

4. It is also useful in determining the meridian of greatest corneal curvature before performing a keratotomy upon the eye. If the meridian of greatest curvature be vertical, a superior or inferior keratotomy will tend to reduce the astigmatism; if the greatest curvature be horizontal, a lateral keratotomy would be preferable.

So many different tests of ametropia have now been

given, that the reader may feel rather perplexed as to which to adopt, and in what order to conduct the examination; the following procedure is suggested.

Procedure.—

1. It will save time first to do a retinoscopy without a cycloplegic, although I personally use cocaine to dilate the pupil.
2. Then test the patient's visual acuteness as on *pp.* 31–40, with the glasses found.
3. Find the position of his near point when wearing the correction.
4. If I cannot readily get his vision up to the standard amount, I at once instil homatropine, unless contra-indicated, and test him again by retinoscopy
5. With young patients, and in all cases of myopia, I always use a cycloplegic.

Practically, I never rely on any determination of a patient's refraction without the instillation of a cycloplegic.

After two or three instillations of homatropine every quarter of an hour, the pupils will be widely dilated, and the accommodation will be practically paralysed in an hour's time. By the next day the effects of the homatropine will have almost passed away. The solution that I use is made according to this prescription:—

℞	Homatr. Hydrobrom.	gr. viij
	Cocain. Hydrochlor.	gr. iv
	Chloretone	gr. ij
	Aq. Destill.	ad ℥j

Chloretone acts as an antiseptic, and prevents fungus from growing in the solution, and cocaine, in addition to its other effects, widens the palpebral opening owing to its stimulation of the sympathetic which innervates Müller's palpebral muscles. This is a great advantage in some elderly people who have a difficulty in raising their upper lids. To avoid any desiccation of the cornea,

it is well to enjoin the patient to keep his lids closed for a few minutes after the instillation.

With young patients, however, in whom I suspect spasm of accommodation, I always use atropine. There are two objections to atropine: (1) It will take about a fortnight for the power of accommodation to be recovered after its use; and (2) There is always a danger of inducing an attack of glaucoma in patients above the age of 40. In patients who have a shallow anterior chamber or increased tension it is advisable for this reason to avoid even the use of homatropine, and to trust to the ophthalmometer, and to retinoscopy with undilated pupils, in association with the usual subjective tests. However, in almost all such cases a moderate dilatation of the pupil can be obtained without risk by the instillation of 1 per cent cocaine or 3 per cent euphthalmine.

There is a great difficulty in obtaining reliable results from a retinoscopy when homatropine is not used. The patient should direct both eyes to the distant wall and, if hypermetropic, he should wear sufficiently strong convex glasses before *both* eyes so that his accommodation should be relaxed as far as possible. If no mydriatic is used, it may be necessary to get the patient to direct his eyes slightly to the left or to the right, according as his right or left eye is being examined, in order to obtain the maximum dilatation of the pupil possible. But it must be remembered that in such a case the surgeon is only attempting to estimate the refraction of the patient's blind spot.

Treatment.—*Hypermetropia.*—When a cycloplegic has been used so that the total H has been determined, the best method is to prescribe lenses which correct all the manifest and one-third of the latent H.

In cases in which the manifest error of refraction has not been determined before the instillation of the mydriatic, it will usually be sufficient to deduct 1 D from the total

H found in the case of atropine, and about ·75 D when homatropine is used.

Should the spectacles be worn *constantly, or only for near work*? So long as $\frac{6}{6}$ can be seen quite comfortably, without any symptoms of asthenopia or headache, I do not insist on the glasses being worn constantly, but I consider them necessary for all near work. When the glasses improve the distant vision, they should be worn constantly.

In all cases of convergent comitant strabismus (C C S), the full correction should be worn constantly, and never more than 1 D taken off for the atropine used.

In the absence of any special instructions to the contrary, it is the duty of the optician to see that the patient's visual lines for distance pass normally through the optical centres of the lenses. It is well for the surgeon to give special instructions for the reading glasses. If the glasses required are the same for both eyes, it is unnecessary to trouble about any decentration downwards, but in all cases as the eyes usually turn downwards about 13° or 14°, the glasses should be tilted 14° downwards. As the eyes are converging towards a point say 30 cm. from the spectacle plane, each eye is converging $5\frac{1}{2}$°, so the visual lines will traverse a point of the glass more than 2·5 mm. nearer the median line than when looking at a distance. It is well to order the optical centres of the lenses to be decentred or displaced 2·5 mm. inwards. Measurement of the actual position of the eyes when converging is apt to be faulty, as the distance apart may be taken to be in the plane of the cornea, and not in the spectacle plane. In the case of anisometropia the surgeon must pay very special attention to the displacement downwards of the optical centres; with this I shall deal presently.

Myopia.—In all cases of high myopia, there is a great tendency for it to increase, so special instructions should

be given to the patients. As the working distance of myopes is usually too close, their eyes are strongly converged and rotated downwards to see the book or the work that is below them. Now, if we think of the muscle that is chiefly employed to turn downwards an eye that is already directed inwards, we remember that it is the superior oblique. When this muscle contracts, especially if there is any synergic action of the inferior oblique, we see that the eyeball must be compressed laterally by these encircling muscles, which have no intra-capsular ligaments to keep off their injurious pressure from the eyeball. It is clear that any lateral pressure will tend to increase the length of the eyeball, and hence to increase the myopia, to cause posterior staphyloma, and even retinal detachment. The work should therefore be raised to the level of the eyes and held as far off as is convenient, say 33 cm. Books should be placed on a suitable book-rest, or held in the hands with the elbows resting on the arms of a chair, and all writing should be done on a sloping desk, with the illumination coming over the left shoulder, so that the letters as they are written may be distinctly seen. In the case of young myopes it is well to consider the advisability of their being educated at the special schools for myopes, of which most satisfactory reports have been published.

Many authorities habitually under-correct myopia. I give the full correction under homatropine, unless the myopia is extreme, above — 15 D, as I find that the effort to see distinctly with an under-correction strains the eyes far more than when the full correction is given, and I have never found any reason to alter my practice when sufficient care is taken to avoid the action of the superior oblique muscle.

Astigmatism.—The whole of the astigmatism found by retinoscopy under homatropine should be corrected by appropriate cylinders, and the glasses should be worn

constantly. If a patient is entirely free from headaches and symptoms of asthenopia, and if his distant vision is only triflingly defective without his spectacles, this rule need not be rigorously insisted on, and he may be allowed to wear them only for close work. When the astigmatism is oblique the correction will be almost always uncomfortable at first; if the discomfort continues for more than three weeks, it is well to examine the case again. Patients will sometimes return complaining of discomfort with their astigmatic correction, when it will be found that their spectacle frames have become bent, so that the setting of the axis is imperfect. All that is necessary in such a case is to send them to the optician to have the frames readjusted.

It is customary to place the spherical correction next the eye, and the cylindrical correction in front when using the trial lenses; but when the optician makes up the spectacles if the spherical component is convex, the spherical surface is often anterior, while the cylindrical surface is posterior. Now, when the astigmatism is of high degree this will involve a slight alteration in the power of the lens. It is well, therefore, when the last stage of the subjective examination is reached, to mount the lenses in the trial frame in the position in which they are ordered to be made; and then try with the crossed cylinder whether any further improvement can be obtained by slightly altering the cylinder and the spherical lens before finally writing the prescription.

After correcting a high degree of astigmatism and obtaining a visual acuteness of $\frac{6}{6}$, or at any rate some of the letters of that line, I have often been disappointed with the result I obtained when testing the patient's near vision. Some three years ago I determined to see if the effective value of the cylinder was altered when used for near work. To my surprise and delight I found that this was so, and my difficulty was thereby explained.

(*Brit. Jour. of Ophthalmology*, Jan., 1925.) I have since found that this work had been previously published in a paper by the late S. D. Chalmers, read before the Optical Society in 1906. On *p.* 189, where the optical principles are explained, I show that if the same refractive error is to be corrected for near work as for distance, the lenses for near work must be increased by about 9 per cent, so that if a man requires 5 D cyl. for distance, he should be given 5·45 D cyl. for work at 30 cm. distance, for this glass will have the same effect at a distance of 30 cm., as a 5 D cylinder has when the incident rays are parallel. Suppose that the patient required + 3 D sph. for distance, a lens of + 3·27 D would be required for correcting this 3 D of hypermetropia if the object viewed were 30 cm. from the spectacle plane and he would still be using $3\frac{1}{3}$ D of accommodation. However, the spherical addition is of much less practical importance than the cylindrical. If, however, a + 1 D sph. is added for presbyopia, an addition to the cylinder of only about 6 per cent is required ; if + 2 D is added there must be an addition of about 3 per cent, but if + 3 D is added no addition will be required. This is readily explained on noting that the effect of adding a + 3 D lens will make the incident rays from the object become practically parallel. By paying attention to this point with regard to the power of cylinders I have lately obtained much better visual results at reading distance.

When the eyes are converged and depressed there is often found to be extorsion, but its amount varies in different cases, it usually gives rise to no discomfort ; but clearly the axis of the cylinder may require altering for near vision. Occasionally I have found that a slight extorsion of high cylinders gives rise to better and more comfortable vision of near objects.

Anisometropia.—In this condition the refraction of the two eyes is different, and three principal varieties are found : (1) Those who have binocular vision. (2) Those

who use their eyes alternately: this is usually the case where one eye is slightly myopic while the other is hypermetropic. Such patients use one eye for close work and the other for distance. (3) Those who only use one eye, the other being amblyopic.

1. In this class the difference of refraction will most probably be small, and when distant vision is improved by the glasses the full correction should be given to each eye. It is commonly stated that if the difference be greater than 1·5 D, the full difference cannot be comfortably worn. This I have not found to be true; after perhaps a fortnight of discomfort, patients are usually delighted with a full correction, even when the difference is 3·5 D or 4 D. The discomfort does not arise from the slight difference in size of the retinal images, for as we have seen in corrected axial ametropia, the size of the retinal image is almost that of a normal emmetrope. If the spectacles were worn at a distance of 15·7 mm. (instead of 13·6 mm.) from the eyes, the retinal images would be exactly the same size in each eye. (*See p.* 167 for proof.) On placing the lenses exactly at this distance from the eyes, it is found that the discomfort is not relieved. The trouble is brought about in this way. Suppose a patient requires R. + 3 D, L. + 6 D, and that, on looking straight forwards with these he sees well. On ranging his eyes from side to side, and especially on looking downwards through the glasses, he will complain of discomfort. This is easily understood when it is remembered that, on looking through an eccentric part of a lens, it will have a prismatic effect that will increase with the power of the lens. The lower parts of convex glasses will of course act as prisms with their bases upwards, and those of concave lenses as prisms with their bases downwards. We are considering now an anisometrope using R. + 3 D, L. + 6 D; on looking at a distant object 15° below the horizontal plane, his eyes will have

to be depressed more than this amount—his right eye 16° 15′ and his left eye 17° 44′—to see the object singly. This is more than he can do, and therefore he complains of a vertical diplopia owing to this difference of level (1° 29′) between the two images (*p.* 211). By using the formula given on that page it will be found that if the book is depressed 15° for reading at a distance of 30 cm. from the plane of the spectacles, the difference of level between the two retinal images will be 15′ less (1° 14′).

It is clear, then, that a fully corrected anisometrope must learn to turn his head and not his eyes; when he has acquired this habit the discomfort will disappear. For reading, unless he be a myope, he must have another pair of spectacles that are properly centred for that purpose. Suppose that his eyes are depressed $13\frac{1}{2}°$, his visual lines will traverse points situated 6·5 mm. below the centres of his glasses. So the optical centres of both his lenses may be lowered 6·5 mm.; this can be done by raising the bridge of his spectacles 6·5 mm. so as to lower the glasses, or by decentring the lenses this amount downwards. A simpler plan will be to decentre only the more convex glass downwards l mm. where $l = \frac{D - D'}{D}$ 6·5 mm. when D′ is the power of the weaker lens. Both lenses with this eccentric vision will act as if associated with equal prisms.

If D = 6 and D′ = 3, $l = \frac{1}{2}(6{\cdot}5) = 3{\cdot}25$ mm. (1·95 ∇ base up). Should the right eye be myopic D′ = − 1, and D = + 2, decentre only the left lens $\frac{2 - (-1)}{2}$ 6·5 or 9·75 mm. down.

The right lens is virtually decentred 6·5 mm. up (i.e. ·65 ∇ base down), and the left is virtually decentred 9·75 − 6·5 = 3·25 mm. down, so it also carries a virtual prism of 2(·325) or ·65 ∇ base down (*see* footnote *p.* 77).

The above does not refer to myopes, who should not be encouraged to depress their eyes. In all cases the lenses should be tilted about 14°, and unless there is esophoria,

the lenses should be displaced or decentred about 2·5 mm. inwards.

As the lateral effect is much less annoying than the vertical displacement, I have sometimes found this difficulty overcome by ordering pince-nez instead of spectacles, as by altering the position of the pince-nez on his nose, the patient can adjust them for reading so that their optical centres are displaced 6·5 mm. downwards, and the lenses themselves have a tilt of 14°.

For a discussion of the prismatic effects of lenses I must refer the reader to the optical section of this book (*p.* 209).

For the exact determination of the anisometropia it is well to employ the method indicated *p.* 139 (13).

2. In these cases we must try to find out what is best for the patient. If provided with a correction, he will in the first place have to learn to use it, and to put up with the initial discomfort, and then he must do stereoscopic exercises for a long time, so as to obtain valuable binocular vision. If the patient is satisfied with his present condition, and if he refuses to train his binocular sense, I merely order the full correction for reading and close work. I have not found one such person with patience enough to persist in his binocular training, to get any result that was worth the time and trouble it cost.

3. Similarly in these cases, unless the patient is young, the training of an amblyopic eye is very tedious, and the rate of improvement steadily diminishes, so that few will be found who will practise their amblyopic eye for the years that are required to get a really satisfactory result. However, this should always be urged. It is a good plan to order the correction and insist on their eating their meals with the better eye covered up. It is important that the exercise should not be too exhausting to the eye, and that it should be regular, frequent, and not too long continued.

Aphakia.—After the crystalline lens has been removed, the eye, if previously emmetropic, requires a strong convex

glass in order to see distinctly. As the refracting system of the eye is now entirely different from that of an ametropic eye, the tests of visual acuteness are no longer comparable with those of normal eyes. Indeed, it is shown on *p.* 179 that the retinal image in a corrected aphakic eye is 24 per cent larger than in a normal eye. Hence vision of $\frac{6}{6}$ in a corrected aphakic would not really correspond to a visual acuteness of $\frac{6}{6}$ in a standard eye.

It is sometimes desirable to know roughly what will be the power of glass required for an aphakic, who previously had a known degree of ametropia. Assuming that curvature of the cornea has undergone no alteration from the operation, and if his previous correcting glass for distance were of D′ dioptres, I have calculated the following approximate formula from what are now the accepted constants of the standard eye. An indication of my method will be found on *p.* 170.

$$D \approx \frac{6{\cdot}59 + {\cdot}4\,D'}{{\cdot}657 - {\cdot}0024\,D'} \; *$$

It will be noticed that according to this formula an emmetrope (where D = 0) who became aphakic should require a lens of power $\frac{6{\cdot}59}{{\cdot}657}$ or + 10·03 D for distant vision, which is in accordance with clinical observation. The formula does not, however, give a satisfactory result for estimating the degree of myopia a patient should have in order that he should see distant objects without any glasses after his lens had been removed. The formula gives $D' \approx -\frac{6{\cdot}59}{{\cdot}4}$ or − 16·5 D ; the experience of English ophthalmic surgeons has been that over − 20 D was required for this result. Richardson Cross published a case some thirty years ago in which he obtained this result after extracting the lens from a myopic eye of − 25 D.

*The sign ≈ denotes approximate equality.

There seems to be some difficulty in estimating the amount of previous myopia required, judging from the very discordant values that have been given by reputed authorities. In his *Eyes and Spectacles* (p. 10) von Rohr tells us " that by a simple calculation an axially myopic eye of $-13\frac{1}{4}$ D is after the operation found to see clearly at a distance ", although he uses Gullstrand's constants for the eye. In the optical section at the end of this book I hope that the correct procedure will be quite clear. The values of the constants now accepted are only approximate average values and cannot be regarded as being correct for every eye. It is perhaps for this reason that the formula given does not hold with anomalous highly myopic eyes.

Tilting the Lens.—It will be noticed that myopic patients often wear their pince-nez inclined, and say that they see better with them than through spectacles which hold lenses of the same power in the normal position. It will be found in such cases that the patients have some astigmatism axis 0°, and by tilting their glasses, they utilize the cylindrical effect of the inclined lens. This is occasionally of service to the practitioner in cases in which the cylinder is weak, with its axis horizontal, and the spherical lens is of high power. I have drawn up the following table, showing the effect of tilting a 10 D lens through different angles. The method of obtaining this table is explained on *p.* 180. The index of refraction is taken to be 1·523.

TILTING SPHERICAL 10 D LENS. $\mu = 1{\cdot}523$.

OBLIQUITY	SPHERICAL	CYLINDRICAL
10°	10·101 D	·314 D
15°	10·228 D	·734 D
20°	10·409 D	1·379 D
25°	10·648 D	2·315 D
30°	10·948 D	3·649 D
35°	11·314 D	5·547 D

It is almost always found that cataract patients, after the operation, require a convex cylinder axis 0°, which in

a few months must be decreased in strength. For hospital patients, it is a great advantage to tilt their spherical glasses downwards. When their astigmatism diminishes, the correction may be simply given by bending the legs of the spectacles upwards. The glasses cost less originally, are lighter, and last longer. When the glass to be tilted is not 10 D, but some other number such as 9 or 11, the figures must be multiplied by ·9 or 1·1, in fact always by the tenth part of the power of the glass.

For instance, suppose an aphakic requires + 11·5 D sph. + 1·5 D cyl. ax. 0°. We see that this result can be nearly obtained by tilting a + 11 D lens 20°,

$$\text{for } 1{\cdot}1 \times 10{\cdot}41 \approx 11{\cdot}5 \text{ D}$$
$$\text{and } 1{\cdot}1 \times 1{\cdot}379 \approx 1{\cdot}5 \text{ D cyl.}$$

We can order him then + 11 D sph. to be inclined downwards 20°.

Suppose a myopic patient of — 20 sph. — 2·5 cyl. ax. 0° presents himself, complaining of the weight of his glasses. If the patient does not wish to go to the expense of getting toric lenses, we shall probably serve him well by ordering — 19 D inclined 20° ; for the figures in the table multiplied by 1·9 show that by this means we obtain the effect of about — 19·78 D and — 2·62 D cyl. ax. 0°.

It is obvious that whenever the eyes range in any direction behind ordinary spectacle lenses the incident rays must strike the lens obliquely, so that there is a slight increase of the spherical power of the lens, and an added cylindrical effect. For this reason, especially with strong lenses, it is necessary to tilt the spectacles 13½° or 14° downwards for reading and close work, in order that the incident rays may strike the lens as normally as possible. But even if the lenses be so tilted, no correction is given for the cylindrical effect induced by the lateral ranging movement of the eyes. It will be noticed that an aphakic patient wearing the ordinary + 14 D for reading is obliged

to move his head or his book for almost each word that he reads. Indeed, as he cannot use the peripheral part of his lenses this may be replaced by a border of plane glass; such glasses are called lenticular lenses and are much lighter.

Periscopic Lenses.—The object of periscopic lenses is to enable the patient to see distinctly when ranging his eyes from side to side, instead of his being obliged to move his head; this is of great advantage to the wearer when playing such games as tennis, or indeed at all times. The problem of designing such lenses has been attacked by many workers for more than thirty years.

In 1898 Ostwalt did some excellent work which first directed my attention to the subject. In 1908 Tscherning showed that for a certain range of convex glasses the astigmatism due to oblique vision through them can be entirely obviated, and that whenever this is the case, there are two forms of meniscus that will serve the purpose; one, a deep meniscus (Wollaston's), the other, a shallower form (Ostwalt's); he also published an elliptical curve from which these two forms of meniscus for any power within this range could be easily read off. Gullstrand about the same time showed that by means of *aspherical* surfaces the range of powers that could be entirely freed from astigmatism was greatly increased. These Katral lenses have to be worked by hand, and are enormously expensive. In 1908 Tscherning published some tables both for distance and for close work (*Recueil d'Ophthalmologie*) of all the powers, 1 to 20 D, both convex and concave, both periscopic and orthoscopic, that would best attain his object. He has published full details of the mathematical method he employed for this magnificent work. I have been unable to obtain the mathematical method employed by Professor Gullstrand; I have been told that it is a patent process.

We have all received advertisements of Punctal and Katral lenses with photographs of test types to show the advantage gained by these lenses in the definition of peripheral parts of the field. I grant that these lenses are far better than my own for photographic purposes, but I maintain that such pictures are no evidence to show that they are useful for visual purposes. The optical conditions are entirely different. The photographic lens is provided with a *fixed* stop; the periscopic lens with a *movable* stop (the

pupil) and so gives a well-defined macular image of the eccentric part of the field under examination when the eye ranges from side to side. The macula is far more sensitive than any photographic plate, even to a thousandth of a millimetre, but at a short distance from the fovea all this extraordinary sensitiveness is lost. M. Dor has taught us that 5° from the fixation point the visual acuteness has diminished to ·25, and at 10° to ·067 of that at the macula. If the eyes be fixed on one letter of the lower part of this page, it will be found that the fourth letter on either side of it cannot be distinguished. Surely to the eye it is a matter of entire indifference whether the peripheral parts of the field are accurately focused or not; they cannot be distinguished in any case.

I gather that most other workers on the subject have directed their attention to correcting all the astigmatism throughout the whole of the field (a solid angle of 60°); in fact they have been designing a miniature photographic lens, except that they have paid little or no attention to the attendant change of power of the lens when the incident rays traverse it obliquely. The method that I have adopted is different, I believe, from that of all other workers on the subject in several details. In the first place the range of *rotation* of the eye is taken to be 30° in any direction from the primary position, it is not the extent of the *field of vision*; indeed, the field is rather greater for concave glasses and less for convex glasses. Then I have calculated the radius of the circle of least confusion given by the meniscus when the eye is directed 30° from the middle line, and I then find *r* the radius of its retinal image. (*See* the tables for convex lenses, *p.* 207.) I endeavour to obtain a retinal image of this circle less than that of a macular cone, the radius of which is ·001 mm. I give the residual uncorrected astigmatism in the column marked As, and I pay due regard to maintaining the back focal distance fairly constant. As suggested above, I pay no attention to the definition of images on peripheral parts of the retina; I have devoted all my efforts to getting the macular vision as perfect as possible when the eye is rotated 30° behind the lens. Further, I have given

the ocular curvature in dioptres that can be ground by the *ordinary tools* in use. The anterior curvature of the lens is given to three decimal places in dioptres as an indication of what should be aimed at ; it is better for the optician to use a tool a trifle lower rather than higher than that which I give unless the difference is very slight. We have all had patients who complained of the curvature of the field when high periscopic lenses were used ; in most if not in all of such cases it will be found that the lenses have too high a power. For instance, for a + 10 D lens on a − 7 D base the anterior curvature will probably be found to be + 17 D. But such a lens has an *effective* power of + 11 D owing to the displacement forwards of the second principal point. The true value for the anterior curvature is d_1 (*p.* 177) where

$$d_1 = \frac{(D_b - d_2)\,\mu}{\mu + (D_b - d_2)\,t} \quad \text{or} \quad \frac{(10 + 7)1{\cdot}523}{1{\cdot}523 + 17({\cdot}00498)} = 16{\cdot}1\text{ D}.$$

I maintain now that the common complaint of curvature of the field is due to this cause ; it has nothing to do with the real curvature of the image field, for in all my recent cases a due diminution of the curvature of the anterior surface has relieved all these symptoms. Full tables of periscopic lenses from 1 to 14 D for both distance and near work are given on *pp.* 207–8.

Now if a periscopic spherocylinder is required, we may encounter great difficulties, which are often insuperable with the tools at present in use. If + 3 D sph. − 4 D cyl. ax. 0° be required in periscopic form, we note that a power of − 1 D is required in the vertical meridian, and a power of + 3 D in the horizontal meridian. The table on *p.* 208 shows that the ocular curvature should be − 7·25 D in the vertical meridian and − 6·5 D in the horizontal meridian. But unfortunately toric lenses are not made on a 6·5 D base, the only toric tools that are in stock are those on a 3 D, a 6 D and a 9 D base, so that

with the available tools the best prescription would be

$$\frac{+\,6\,D \text{ ax. } 0^\circ + 10\,D \text{ ax. } 90^\circ}{-\,7\,D}$$

But this would not be perfectly periscopic up to the standard solid angle of 60°. If no toric surface is ordered, the best form for the prescription would be — 1 D sph. + 4 D cyl. ax. 90° with the concave spherical surface next the eye. As it must be remembered that lateral movements of the eye are required much more than vertical movements it is generally advisable to endeavour to obtain the greatest concavity in the horizontal direction next the eye. For instance, if the prescription were + 3 D sph. — 4 D cyl. ax. 90° and the concave cylindrical surface were placed next the eye, the periscopic result, although by no means good, would satisfy most patients. A solid angle of 60° is a very generous allowance; the printed line of this book when held at 32·7 cm. from the centre of rotation of the eye, or 30 cm. from the spectacle plane, only requires a range of movement less than 8° to either side of the middle line, i.e., less than a solid angle of 16°. In the case of reading glasses adjustment has been made in my Table as far as possible for viewing a flat object, in which case its lateral parts are obviously more distant from the eye than its central part. D′ gives this eccentric power. (*Brit. Jour. of Ophthal.*, July, 1926.)

Bifocal Lenses.—The upper parts of these lenses are adapted for distant vision, while the lower parts are adapted for reading and close work. In the original form, "the straight split bifocal," or "Franklin," the line dividing the two lenses is horizontal and in the middle. In the "Kryptok" a concavity is ground in the lower part of the distance-glass, in which is fused a glass of higher refractive index; the whole surface is then ground to the requisite curvature, and the lower portion being of higher refractive index, has the required higher power

for reading. These Kryptok lenses are very ingeniously made, but if the reading addition required is over + 2 D, a glass of such a high index of refraction is used that patients complain of the chromatic aberration induced. There are several other kinds of bifocal lenses on the market, which it is unnecessary to specify. The most usual form is that in which a thin convex wafer is cemented on to the lower part.

Great credit is due to the optician for making such artistic spectacles, but many will be found to fail in practice, as some of the essential requisites have not been fulfilled. It will be well, therefore, to consider in detail what these requirements are.

1. *Size.*—The smaller the reading segments are, the greater will be the field of view for distance, so we must determine how large it is necessary for them to be. I have long insisted that they are usually made much too large, and of the wrong shape.

No one will care to see a line of print more than 6 in. (152 mm.) long without moving his head; indeed, unless the glasses were periscopic, he would only see indistinctly on turning his eyes behind his glasses more than this amount. Now the distance (k) of the centre of motility of the eye from the lens is 27 mm., and if we assume the work to be held at a distance of 327 mm. from the centre of motility, it is clear that the reading wafer need never be wider than $\frac{152}{327}$ of 27 mm., say 13 mm., nor greater in height than $\frac{102}{327}$ of 27 mm., say 8·5 mm., unless the patient wishes to have a vertical range of more than 102 mm. (4 in.) when reading without moving his head. Theoretically, the upper margin of the reading portion should be a horizontal line, and not the convex curve that is usually seen. We may say, then, that the reading portion should be rectangular in shape and rarely need be more than 13 × 8·5 mm. in size. These dimensions will enable the wearer to have a much larger field of view for distance.

2. *Position.*—In isometropic cases the wafers may be normally centred and so placed that their upper margins are 1·5 mm. below the mid-horizontal line of the distance-glasses, and displaced 2·5 mm. inwards, so that the eyes when converging may utilize the whole lateral range allowed by the entire width of the wafer. Some patients prefer the wafers to be set one or two millimetres lower down.

Many people when wearing bifocals complain of a difficulty in going downstairs, as the steps below appear blurred, for they can only see them through the reading segments. Such a difficulty is easily obviated by not allowing the segment to extend to the bottom of the spectacle frame. It will be found that if a narrow strip 3 mm. wide of the distance-glass at the very bottom be uncovered by the rectangular reading wafer, a view will be obtained of several steps below. As the ordinary spectacle glass is 37 mm. × 28 mm., a rectangular wafer 8·5 mm. in height, if cemented as described, will allow the wearer to see several feet of the ground in front of him.

3. *Optical Centre.*—In anisometropia the optical centres of each distance lens are placed opposite the pupils of the eyes in the primary position; but when the eyes are depressed for reading as explained on *p.* 66 there will be a difference of level between the two retinal images. It is necessary to decentre the reading wafers in such a way that on looking at the print through the reading combination the retinal images shall be on the same level, or fall on the corresponding points of each retina. This is most important, as normally it is impossible for one eye to turn up and the other to turn down. The fact that bifocals are so commonly unsatisfactory to anisometropes is always due to carelessness in this particular. It is quite unnecessary to place the optical centre of each reading combination 6·5 mm. downwards; though this would correct the error, it would necessitate the lower

borders of the wafers being needlessly thick in cases of hypermetropia, and the upper borders in those of myopia. A better correction can be made with the help of this simple formula :

$$V = \frac{D - D'}{d} \left(\frac{h}{2} + 1{\cdot}5\right) \text{ mm.*}$$

Where D represents the stronger lens, and D′ the weaker lens and d is the power of the reading wafer, while *h* is its height (8·5 mm.), then V gives the required decentration of the wafer that is attached to the stronger lens. Take, for instance, the previous case of anisometropia, Right + 3 D, Left + 6 D, and suppose that the wafers are of power + 2 D.

$$V = \frac{6 - 3}{2} \, 5{\cdot}75 \text{ mm.} = \frac{17{\cdot}25}{2} = 8{\cdot}625 \text{ mm.}$$

The right wafer is to be normally centred, and the left wafer must be decentred downwards 8·6 mm. In previous editions I used a notation of signs to indicate whether the decentration should be upwards or downwards, but this method seemed to give rise to endless confusion. It is the simplest way to regard the matter from a common-sense point of view. The patient is looking through the lower parts of *convex* lenses ; they therefore act as prisms with their bases *up*, the stronger prism must be corrected by a prism with its base down, which is effected by decentring the convex wafer downwards. When each eye is depressed so that its visual line traverses a point 5·75 mm. below the mid-horizontal line of the spectacles the virtual prisms before each eye will be the same. The centre

* This formula is not absolutely accurate, as it assumes that the increased depression of the left eye is identical with the increased deviation of the left prismosphere. But on *p.* 214 it is shown that this is not the case ; however, on doing the calculation involved, it will be found that the above formula gives a result that is accurate to a fraction of a millimetre.

of the left wafer, which is supposed to be 8·5 mm. in height, is 4·25 mm. below its upper margin, or 5·75 mm. below the level of the centres of both distance-glasses. If the wafers were of some different height, say 9 mm., the geometrical centre of each wafer would be 6 mm. below that of the centres of the distance lenses. In that case

$$V = \frac{6-3}{2} 6 = \frac{18}{2} = 9 \text{ mm.}$$

When the distance correction is spherocylindrical, the power of the lens varies with the meridian considered; clearly for the above one must find the power in the vertical meridian. For instance, + 1 D sph. + 4 D cyl. ax. 90° is of course + 1 D in the vertical meridian, but + 5 D in the horizontal meridian. A cylinder of power D set at an angle θ gives the power D $\sin^2 \theta$ in the horizontal meridian, and power D $\cos^2 \theta$ in the vertical meridian. A 4 D cylinder set at an angle of 60° has the power $4\ (\tfrac{1}{2})^2$ or 1 D in the vertical meridian, and $4(\tfrac{\sqrt{3}}{2})^2$ or 3 D in the horizontal meridian. With most angles, however, some tedious arithmetical work may be required in squaring a decimal. All this will be saved by remembering that

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}, \text{ and } \sin^2 \theta = \frac{1 - \cos 2\theta}{2}.$$

As an example, suppose that it is required to find the power of a − 4 D cyl. ax. 25° in the vertical meridian; in the table, *p.* 235, we see that cos 50° = ·64279,

$$\text{so } \cos^2 25° = \frac{1{\cdot}64279}{2} \therefore -4 \cos^2 25° = -3{\cdot}28558 \text{ D}$$

which is the power of the cylinder in the vertical meridian. In actual practice we should take it as − 3·3 D, at any rate never more than two decimal places need be considered.

I very strongly recommend the cemented form of bifocal just described; in other forms such as the unibifocal, in which the reading segment is actually ground on the

surface of the distance lens, it is extremely difficult and sometimes impossible to correct for the difference in level in cases of anisometropia. If the corners of the rectangular wafers are rounded off, they present quite an artistic appearance.

I do not recommend bifocal lenses for myopes unless of low degree, for, as I have previously explained, I always warn myopes against turning their eyes downwards when converging. In two years I have had two patients with a myopia of less than -6 D who acquired a detached retina after wearing bifocal lenses. I am inclined to think that the bifocal glasses had something to do with the cause of the disaster.

Peripheral Vision.—The visual acuteness for form described on *p.* 26 *et seq.* refers to direct macular vision ; indirect vision for form is far less acute, as the eccentric parts of the retina are structurally different from those of the macular area. If the visual acuteness for the fixation point be regarded as 1, M. Dor found the following values for the visual acuteness for objects at the given angular distance from the fixation point :—

5°	10°	15°	20°	25°	30°
$\frac{1}{4}$	$\frac{1}{15}$	$\frac{1}{30}$	$\frac{1}{40}$	$\frac{1}{50}$	$\frac{1}{70}$

When dealing with periscopic lenses (*p.* 72) I drew attention to the indistinctness of peripheral vision, and urged this in support of my method of treating that problem. But it must not be inferred that peripheral vision is therefore of little value. When walking on the pavement one avoids bumping against other people, lamp-posts, and other obstacles entirely by eccentric vision ; it is not necessary to look directly at the obstacle, for one's sense of direction (or projection, as it is called) of an object in the peripheral part of the field is extraordinarily acute.

Freeland Fergus has shown us that if a mark be made on a sheet of paper and a small coin, a shilling or a halfpenny, be placed 17 cm. from it, and the gaze be fixed on the mark at a distance of 30 cm. the coin can always be struck with a pencil. In this case the image of the coin is formed on a peripheral part of the retina 30° away from the fovea, and yet the visual acuteness for form is only $\frac{1}{70}$ of that at the fovea. Indeed, he asserts that nearly all kinds of manual labour depend far more on good peripheral vision than on good central (macular) vision. It is noteworthy also that a moving object in the periphery of the field at once calls one's attention to it, although the image of a stationary object in the periphery will be neglected if one's attention is directed elsewhere, as in reading. Motor accidents, if not due to carelessness, are more frequently due to a defect of the field of vision than to any defect of macular vision. All that is necessary is to see that there is an obstacle; it is quite unnecessary to define it. Peripheral vision appears to depend almost entirely on "light sense," not on "form sense," or as many contend with good reason on "rod vision" and not on "cone vision." But it is not the province of this book to discuss the vexed question of light sense. It will be noted that the main purpose in daily life of peripheral vision is to enable us to avoid accidents, and to inform us of the position of objects towards which we must direct our eyes if we wish to examine their details—in fact, to tell us where to look. The importance of peripheral vision can hardly be overestimated in the case, for instance, of a seaman on watch, and yet no examination of his field of vision is required, although the tests of his macular vision seem to me to be unnecessarily severe.

CHAPTER III.

FAULTY TENDENCIES AND DEVIATIONS OF THE OCULAR MUSCLES.

PRISMS.

As prisms are so largely used in the diagnosis and in the treatment of these faulty tendencies, it is advisable first to describe their clinical properties. They have several other interesting physical properties which it is not necessary to specify, as the ophthalmic surgeon makes no use of them. Indeed, the prisms he employs have such an acute angle that their only property which concerns him is that of altering the direction of the incident light.

A square or rectangular prism is a wedge of glass presenting two rectangular "refracting surfaces" which meet at an acute angle called the "apical angle." The line of junction is called the "edge," and the opposite part or "base" presents a narrow rectangular surface.

If such a square prism be trimmed into a circular or elliptical shape, the "edge" is the tangent to the thinnest part, and the "base" is the tangent to the thickest part. The line that passes through the centre at right angles to these two tangents is the "base-apex" line. In prisms of elliptical shape for spectacle frames, if the base-apex line is oblique, the thinnest part of the glass will not represent the apex, nor will the thickest part represent the base of the prism, but the base-apex line will be that drawn through the centre at right angles to the two parallel tangents at the thickest and thinnest parts of

the glass. In fact, if the base-apex line be oblique, it will be less inclined to the vertical than the line joining the thickest and thinnest parts of the glass.

The path of light transmitted through a prism is always deviated towards the base of the prism. Consequently an object when viewed through a prism will always appear to be displaced towards the edge of the prism, for the direction in which an object is seen is that which those rays of light that come from it have, just before entering the eye. The amount of this deviation depends primarily upon the apical angle of the prism and upon the refractive index of the glass of which it is composed, although it depends, to some extent, upon the tilting of the prism with reference to the incident light.

The glass most commonly used in England for spectacles has a refractive index (μ) equal to about 1·523 ; this is the standard value of μ in America. It is found that the deviation induced by a prism is a minimum when the light passes symmetrically through it, or when the angle of incidence is equal to the angle of emergence,* in other words, when the path of light in the prism is parallel to the base of the prism. When all the angles concerned are small, the approximate formula, $\delta = (\mu - 1)$ A, may be regarded as true,* where A denotes the apical angle of the prism and δ its deviation. When A is not greater than 10°, even when the angle of incidence is as much as 20°, the formula gives a result well within 5 per cent of the truth. We shall assume, therefore, for practical purposes that this formula gives the true deviation of any prism that is used by an ophthalmic surgeon.

Nomenclature.—There are now six different methods of denoting prisms, and hence some confusion has arisen. It will be necessary to describe all the methods, and I shall definitely give the reasons for my own preference.

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* For proof, *see* my *Geometrical Optics*, p. 44–5.

1. The prisms in most trial cases are marked with the degrees of their *apical angles*.

This method has nothing to support it except the convenience of the optician. Prisms are used for the *effect* that they produce, and they should therefore be named according to this effect, just as lenses are named according to their power. It is useless to know either the curvature of the surfaces of a lens or the apical angle of a prism, unless we also know the refractive index of the glass used. Even when we know μ it requires a small calculation to find its deviation, and then, in the prescription to the optician, it would be necessary to give the apical angle as well as the μ of the glass required for the prism.

2. The prisms are indicated by their angles of *minimum deviation*. This is an excellent method, and in prescriptions is always denoted by the small letter *d*. Thus a prism of 3° *d* means a prism that exerts a minimum deviation of 3°, and it is left to the optician to make it up of any crown glass he pleases, according to the formula $A = \frac{\delta}{\mu - 1}$; thus if his glass has $\mu = 1{\cdot}54$ the apical angle must be about 5° 33′, if $\mu = 1{\cdot}52$ the apical angle must be 5° 46′. It does not matter to the surgeon or to the patient which is supplied him, as long as the desired effect is produced.

3. *The Metre Angle.*—This is the most convenient unit for plotting charts of convergence, but it is ill adapted for the measurement of prisms. It is the angle (NMO) through which each fixation line sweeps in moving from parallelism to view a point in the middle line one metre distant from the centre of motility of the eye (*Fig.* 8). The angle that the fixation line of each eye passes through in viewing a point ½ m. distant is 2 ma. (two metre angles), and so on. Nagel devised this method so that the number

of metre angles of convergence exerted by each eye should be numerically equal to the number of dioptres of accommodation exercised by an emmetrope in viewing the point at the given distance. Thus when reading at the distance of $\frac{1}{3}$ m., it was supposed that 3 ma. of convergence

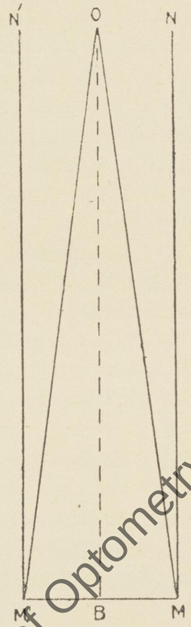


Fig. 8.

and 3 D of accommodation were exercised by each eye, or at any rate whatever difference there was between them could be neglected with impunity. In the above case where OM = $\frac{1}{3}$ m., if the point of fixation be $\frac{1}{3}$ m. from the centre of rotation, it is only 30·63 cm. from the spectacle plane at L, for LM $\approx$ 2·7 cm., and would, therefore, require a lens of 3·26 D to be worn to correct it. Of course, if the

accommodation exercised in this special case be measured by the distance of the first principal point of the eye from the point of fixation (OH′ = 32·13 cm.), the accommodation would be denoted by 3·11 D. For higher degrees of convergence the discrepancies become greater: e.g., for 6 ma. OL = 13·97 cm. the accommodation exercised would be 7·16 D, or if the first principal point of the eye be regarded as a more suitable reference point OH′ = 15.46 cm. and the accommodation exercised would be estimated to be 6·47 D.

There are also two other weighty objections to it.

a. Its size is different in different individuals, for clearly it depends on the interocular distance (*Fig.* 8). The angle NMO = BOM = $\sin^{-1}\frac{BM}{OM}$. Thus, when OM = 1 m., if M′M, the interocular distance, be 58 mm., BM = 29 mm., one metre angle (1 ma.) has the value $\sin^{-1}$ ·029 or 1° 40′; but if M′M = 64 mm., BM = 32, so 1 ma. = $\sin^{-1}$ ·032 or 1° 50′. (For M see *Fig.* 9, *p.* 115.)

b. There is a difficulty with multiplication, for 5 ma. is not five times the size of 1 ma.; if 1 ma. = $\sin^{-1}$ ·032 or 1° 50′, 5 ma. = $\sin^{-1}$ ·16 = 9° 12·4′, not 9° 10′.

The metre angle unit may be entirely neglected unless charts such as those shown in *Figs.* 10–14 are being drawn up.

4. *The Centune or Percentage System.*—We owe to Dr. Maddox this very convenient term for naming a class which contains three subdivisions, but the units of all of them are one percentage angles, that is to say, the unit prisms all produce a deviation of 1 cm. at a distance of 1 m.

The unit *arc centune* is the angle subtended at 1 m. by 1 cm. of arc; in the *tangent centune* the length of one centimetre is measured on the straight line perpendicular to the base of 1 metre's length, while in the *sine centune*

the metre's length is measured along the slanting radius vector, and a perpendicular of 1 cm. is dropped from the extremity of this radius vector to the base.

Only the first two of these centune angles need be considered; they have been used under other names for many years.

a. Prism Dioptre or Tangent Centune.—This unit was proposed by Mr. C. F. Prentice, and is designated by an upright delta ($\triangle$). One prism dioptre is therefore the angle whose tangent is ·01 or 34·37642′, 10 $\triangle = \tan^{-1} \cdot 1$ or 5° 42·6355′, it is not equal to ten times the value of 1 $\triangle$. This is a great disadvantage to the unit. Its charm consists in the delightful simplicity that is brought into problems of decentration by its use. Due credit is rarely now given to Mr. Prentice for his ingenious innovation. The prismatic effect of decentring a lens is represented by Dc, where D is the dioptric power of the lens, and c is the decentration in centimetres. Thus a + 3 D lens decentred $\frac{1}{2}$ cm. outwards is equivalent to a + 3 D lens associated with a prism of $1\frac{1}{2}$ $\triangle$ base outwards. If of course it is decentred inwards, it is equivalent to a prism base inwards. No thought is required; the thickest part of a convex lens is its centre, so the direction of the decentration must refer to the base of the prism. If the lens considered be a concave one, the *minus* sign reminds us that the decentration will refer to the position of the thin edge of the prism. A − 2 D lens decentred $\frac{1}{2}$ cm. *upwards* is equivalent to a − 2 D lens associated with a 1 $\triangle$ prism edge upwards or *base downwards*.

b. Centrad or Arc Centune.—This unit was proposed at a later date by Dr. Dennett, and is the hundredth part of a radian, i.e., ·5729578° or 34·37747′, and is denoted by a reversed delta ($\triangledown$). With this unit there is no difficulty about multiplication, e.g., 10 $\triangledown$ is ten times one centrad. The difference between the two units is exceedingly minute, and, on examination, the apparent superiority of the prism

dioptre for decentration problems is found not to hold. For, as Maddox has pointed out, owing to spherical aberration, the effect of decentration is rather more than is indicated by the tangent formula. Hence, as with the prism dioptre, when N denotes the number of centrads, $N = Dc$, or $\frac{Dl}{10}$ where l is the number of millimetres that a lens of power D is decentred ; and of course $l = \frac{10\,N}{D}$.

A convex lens of $+ 5\,D$ combined with an *adducting* prism of $2\,\triangledown$ (i.e., base out) is equivalent to the convex lens decentred *outwards* 4 mm.

If when testing a patient one has been using prisms marked by their apical angles (A), it is useful to know that $l \approx \frac{11\,A}{D}$. Thus if in the above case the prism required were 2 A, the $+ 5\,D$ lens should be decentred 4·4 mm. outwards.

The relation between the unit centrad ($\triangledown$) and the unit metre angle (M) may be fairly accurately expressed by the following formulæ, where m denotes half the interocular distance in millimetres,

$$\triangledown \approx \frac{10\,M}{m} \quad \text{and} \quad M \approx \frac{m\,\triangledown}{10}$$

or $M \approx 3\triangledown$ when $m = 30$ mm. It must, however, be remembered that we cannot use this formula if we have to multiply metre angles by high numbers ; even 10 M is not $30\triangledown$, but $30{\cdot}47\triangledown$

$$\text{for } 10\ M = \sin^{-1} {\cdot}3 = 17^\circ\ 27'\ 27{\cdot}4'' = 30{\cdot}47\triangledown.$$

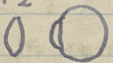
Again, as $4^\circ = 6{\cdot}981\ \triangledown$ (*Table III, p.* 235),

$$1^\circ \approx 1\tfrac{3}{4}\ \triangledown \quad \text{and} \quad 1\ \triangledown \approx \tfrac{4}{7}^\circ$$

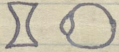
The American Ophthalmological Society has adopted the centrad as its official unit, and although in this country it makes its way but slowly, I hope it will be universally

With + 2 before the eye the P.R. is at 40", with a - 2 the P.P. is at 10".

+2

40  1 D. H.

-2

20" 10"  1 D H.

-2 -1 - 4 = 7 D Amp.

S. R. 1 D. Hyp.
Amp. 7 D.
P.R 40" Neg.
P.P. 6.5 Pos.
Range. 40" N to 6.5" P.

Add for 2 D Pres. +2
Total Reading +3.

With a +3 before the eye the P.R. is 20" Neg with a +1 before the same eye the P.P. is at 13" Find Range of Acc of Naked eye.

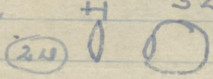
+3

20" or 2 d.

+3 + 2 or 5 D H.

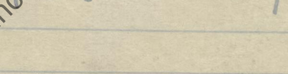
5 D H.

13"



5 + 2 = 7 D Amp

20"



8"

Range of Naked Eye.

adopted before very long. For its many conveniences I think it is far the best unit that has been devised.

c. Sine Centune.—A railway gradient of one in a hundred would be an angle of one sine centune or $\sin^{-1} \cdot 01$ or $34 \cdot 37815'$. It will be seen that it is a trifle greater than one centrad and the prism dioptre, but there is the same difficulty in multiplying it that we found with the prism dioptre, and the metre angle.

The only advantage of the *sine centune* (Σ) is that half the inter-ocular distance in centimetres gives the number of sine centunes in the unit metre angle for that patient exactly, or if m denotes half the inter-ocular distance in millimetres,

$$M = \frac{m\Sigma}{10} \text{ and } \Sigma = \frac{10\,M}{m}$$

It will be noticed that one centune of each of these three varieties may be regarded as equal to $34 \cdot 377'$, and that, therefore, the distinction between them may be thought unnecessary. The difference between them, however, is very apparent when a hundred centunes of each variety is considered. Thus $100 \,\triangle = 45°$,

$$100 \,\nabla = 57 \cdot 296°, \text{ but } 100\,\Sigma = 90°.$$

Resultant Prisms.—We sometimes wish to give a correction for a faulty tendency upwards as well as inwards or outwards; in such cases it is necessary to know how to correct both faulty tendencies of the eye with a single prism. Let θ be the prism required to correct the horizontal deviation, and let ϕ be that for the vertical deviation, then both errors will be corrected by a prism of deviation δ set at an angle ρ with the horizontal line,

$$\text{if } \tan^2\delta = \tan^2\theta + \tan^2\phi, \text{ and if } \tan \rho = \frac{\tan\phi^*}{\tan\theta}$$

As the prisms that can be worn continuously are of such

* *See* my *Geometrical Optics*, p. 47.

feeble strength, no great error is introduced by replacing the tangent of the angle by the angle, and hence Dr. Maddox's simple device is practically quite accurate enough. Draw a horizontal line BA as many centimetres long as there are centrads (or degrees) in the horizontal prism ; let B represent its base, A its apical edge. On A erect a vertical line AE as any centimetres long as there are centrads (or degrees) in the vertical prism if its edge points upwards ; if its edge points downwards, draw AE downwards. Join BE ; its length in centimetres denotes the power of the resultant prism in centrads (or degrees), while the angle at which the prism should be set is the angle ABE between the base BA and the hypotenuse BE.

If a patient required a prism $\nabla_1 = 3\,\nabla$ base down before the right eye, and a prism $\nabla_2 = 4\,\nabla$ base out, it is clear that as

$$R = \sqrt{\nabla_1{}^2 + \nabla_2{}^2} = \sqrt{3^2 + 4^2} = 5\,\nabla$$

a single prism of $5\,\nabla$ set at the angle $\tan^{-1}\frac{3}{4}$ or 36° 52′ with its base pointing downwards and outwards before the right eye would do all that is required. It would be far better to give a prism of half the power to each eye, or $2\frac{1}{2}\,\nabla$ set at 36° 52′ with its base pointing downwards and outwards before the right eye, and another $2\frac{1}{2}\,\nabla$ with its base pointing upwards and outwards at 36° 52′ before the left eye. When writing the prescription an oblique line indicating 37° should be drawn through both the diagrams for the right and left eyes, writing " base " at the outer and lower extremity of the line for the right eye, and " base " at the outer and upper extremity of the line for the left eye. If the patient had required $3\,\nabla$ *base down* and $4\,\nabla$ *base in* before the right eye, the oblique lines in the diagrams should indicate the axis 143° (i.e., 180° − 37°), and the word " base " should be written on the inner and lower extremity of the oblique line for the right eye,

and on the inner and upper end of the line for the left eye.

Should we wish to determine the effect of an oblique prism in the vertical and horizontal directions, we have

$$\nabla_1 = \nabla \sin \rho, \text{ and } \nabla_2 = \nabla \cos \rho \quad (\textit{Table, p. } 235.)$$

In the case above $\nabla_1 = 2{\cdot}5\,({\cdot}6) = 1{\cdot}5\,\nabla$,

$$\text{and } \nabla_2 = 2{\cdot}5\,({\cdot}8) = 2\,\nabla,$$

so we have given a correction for a relative elevation of the right eye of 1·5 ∇ and a relative depression of the left eye of the same amount (1·5 ∇), as well as correcting the lateral defect by 2 ∇ before each eye.

Clinical Properties of Prisms.—The position of objects as seen by the eye depends upon the direction in which the light from the object enters the eye. Hence, if a prism be held before one eye, the other being closed, the light that enters the eye will have been deflected towards the base of the prism and consequently the eye will turn towards the edge of the prism in order to receive them. If the object viewed be distant, it will appear to be displaced towards the edge of the prism in such a way that its angular displacement will be equal to the angle of deviation.

When both eyes are used and a prism is held before one of them, the result is different.

A. When the object is distant.

1. If the prism be too strong for its effect to be overcome by the ocular muscles, diplopia will result. If, for instance, when both eyes are viewing some distant object, a prism causing 5° of deviation be held before the right eye, base in, two objects will be seen; the left eye will see the object in its true position, but the right eye, being unable to turn itself outwards to this amount, will judge from the direction of the entering rays of light that the object has been displaced to the right by an angular displacement

varying from about 5° to 3°. The images will seem to approach each other with every attempt of the eye to overcome the effect of the prism.

2. If a weaker prism, say one of 3°, be used, as the ocular muscles can diverge to this extent, only one object will be seen, but it will appear to be displaced 1½° towards the edge of the prism. Normally the eyes can diverge 4°, i.e., each eye can rotate outwards 2° from parallelism, so that when a 3° prism is held before the right eye, base in, each eye diverges 1½°, and then a ranging movement of both eyes 1½° to the right, enables the image of each eye to be formed on the macula. As the direction of an object when viewed binocularly is judged from the cortical innervation of the ranging centre, the object is assumed to be 1½° to the right.

B. When the object is close to the observer. The displacement is not so great as the deviation of the prism, even when only one eye is used, as can be easily seen by drawing a diagram. The deviation of the prism is indicated by the ratio of the arc of displacement of the object to its distance from the prism; whereas the deviation of the eye when only one is used is indicated by the ratio of the arc of displacement of the object to its distance from M, the centre of motility of the eye.*

1. If a prism, too strong to be overcome by the ocular muscles, be used as before two objects will be seen, whose

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* Let L denote the arc of displacement, p the distance of the object from the prism, $p + k$ the distance of the object from the centre of motility of the eye. Then if δ denote the deviation of the prism, and θ the deviation of the eye,

$$\delta = \frac{L}{p} \text{ and } \theta = \frac{L}{p + k}$$

$$\therefore \frac{\theta}{\delta} = \frac{p}{p + k} = 1 - \frac{k}{p + k}$$

When the prism is in a spectacle frame worn in the usual position, $k = 27$ mm., a little more than an inch.

distance apart varies with each attempt of the eyes to overcome the prism.

2. If a weaker prism be used, the object appears to be displaced through half the displacement that occurs when the other eye is closed. The object will also appear to be closer to the observer when *adducting* prisms (bases out) are used, but further off when the experiment is made with *abducting* prisms (bases in). This is due to the fact that the distance of an object is estimated almost entirely by the cortical innervation of the converging centre. When the object is close, some estimate of its nearness is made from the amount of accommodation exercised to see it distinctly; in this case the two functions are opposed, and so the position assigned to the object is not quite that due to the estimate given by the convergence exercised. With adducting prisms the object appears to be closer owing to the increased cortical innervation of the converging centre, it also appears smaller for the following reason. If an object is brought to two-thirds of its original distance from the eyes, its retinal image will be about $1\frac{1}{2}$ times greater; but in this case the retinal image is of exactly the same size, so the judgement is made that the object is smaller.

It is an interesting point, as it shows how far more important in the estimation of distance is the convergence than the accommodation exercised. The size of a retinal image taken by itself is a very unreliable guide to the real size and distance of an unknown object. The sun subtends an angle of about 32′, yet at sunset it seems to be a great deal larger than when at the zenith; it is hard to realize that its retinal image is smaller than that of a halfpenny at the distance of 100 in., which subtends an angle of 1 ∇, and is far smaller than that of a threepenny bit held at arm's length.

Prisms are often used in the consulting-room to determine the amount of convergence and of divergence that can be exercised by the eyes when they are focused for a given point. As we shall see, it is important in many cases to find this range of convergence when the

accommodation is exercised for reading distance, and again when the accommodation is relaxed. In this case it is obvious that the prisms do not test the strength of the muscles, but the *relative* range of convergence.

The practical points which result from this discussion may be briefly summarized thus :—

1. The effect of a single prism before one eye is equivalent to the effect of two prisms of half the strength, one before each eye.

Of course, if horizontal deviations are under consideration, the two prisms must be placed both bases inwards, or both bases outwards, as the case may be ; when vertical displacements are being dealt with, one must be placed base upwards, the other base downwards.

As chromatic and prismatic aberrations are minimized when weak prisms are used, it is always better to prescribe a weak prism for each eye rather than one strong prism for one eye.

2. Abducting prisms are placed bases inwards ; adducting prisms bases outwards.

3. *Relieving Prisms.*—For the relief of diplopia, prisms, if ordered, must be placed so that their edges are in the direction of the deviation. Thus, if the eyes deviate outwards, the edges of the prisms must be placed outwards. In other words, the bases of the prisms must be on the side of the inefficient muscles. For the relief of divergence, or heteronymous diplopia, use *abducting prisms* or prisms with their bases inwards. For the relief of convergence, or homonymous diplopia, use *adducting prisms* or prisms with their bases outwards.

4. The adequacy of muscles is tested by noting the extent of their ranging movements. The relative range of convergence and supravergence is tested by the strength of the prisms that can be overcome.

The following simple experiment proves that the estimation

of the direction of an object is due to the innervation of the "ranging centre," while the estimation of its distance is due to the innervation of the "converging centre." It is described in Maddox's book, *The Ocular Muscles*, p. 89. Place a tiny piece of white paper on the black cover of a book, and holding it about 6 inches from the eyes look at the piece of paper intently. Now suddenly cover the right eye. The paper will appear to move to the right, and also to recede to a distance.

At first, both eyes were converging to a point at a distance of 6 inches; on covering the right eye it is found to be less fatiguing to the nervous system to relax the convergence exercised and to innervate the ranging centre to the right. As a proof of this, the right eye will be seen to move to the right behind the cover. The object, therefore, appears to move to the right owing to the ranging innervation to the right, and it appears to recede owing to the diminution of the convergence innervation. It is not due to any afferent impulse of "muscular sense" from the state of tension of the muscles in the left eye; for, as the left eye remains absolutely fixed in position, the tension of its muscles is unaltered.

DEVIATIONS.

These may be either manifest (i.e., squints) or latent. We owe to Dr. Stevens the term *heterotropia* to include all manifest squints, and *heterophoria* to include all latent deviations, i.e., those deviations which only manifest themselves when single binocular vision is rendered impossible. *Orthophoria* is the term used for perfect muscular balance. The subjective tests of these conditions are the more important, as they admit of much more accuracy and refinement than the objective tests; although we may be sometimes misled by faulty statements of the patient. We shall consider vertical deviations first, as they are the easiest to understand and to correct.

The terms *hypertropia* and *hyperphoria* denote an upward deviation or an upward tendency respectively. Now, except in paralytic cases, it is generally impossible to determine which eye is at fault, so that the term right hyperphoria means that the right eye tends to assume

a direction above that of the left, and it is fortunately unnecessary, for purposes of treatment, to determine whether the right eye tends upwards or the left eye tends downwards. It is always important to test for comitancy, i.e., whether the deviation remains the same in all directions of the eyes. Vertical deviations give rise to much greater discomfort than convergence defects of the same amount, so that it is most important to examine the vertical equilibrium as accurately as possible before proceeding to the examination of the convergence function. The first case of hyperphoria I ever recognized, now some thirty-five years ago, puzzled me extremely, because I found the convergence range so defective. Eventually I discovered the hyperphoria, and found, on correcting this with a suitable prism, that the convergence range was then perfect.

Symptoms.—The most characteristic symptom of heterophoria, though generally absent, is periodic diplopia. The unconscious effort to overcome a heterophoria in the interests of single vision may occasion asthenopia, headache, which is usually occipital, giddiness, and all the symptoms of depression one finds in neurasthenia. When the tired nervous centres give up the effort to maintain single vision, diplopia ensues, and possibly a visible squint; this, however, usually lasts for only a short time, and then single vision is re-established. The giddiness due to heterophoria may be easily recognized by this peculiarity, that it ceases when one eye is closed. Patients will often acquire the habit of temporarily closing one eye; indeed, the facial muscles of one side of the face are sometimes, especially in hyperphoria, affected with nervous twitching movements that are entirely cured with an appropriate prismatic correction. In several cases for which I have ordered prisms I have been told that there has been a remarkable change of disposition; previous moroseness and pessimism have been replaced

by geniality and optimism, but of course such a fortunate result cannot always be expected. I have already alluded to the anomalies of convergence induced by hyperphoria. There is often present a tendency to convergence (esophoria) at a distance, but a tendency to divergence (exophoria) at reading distance.

In hyperphoria I have sometimes found a peculiar but characteristic symptom, the inability to read near type (J_1) with both eyes, although each eye separately can read J_1 quite easily. No such difficulty is found with the distant type; $\frac{6}{6}$ is read fluently with both eyes together, and yet there is no defect of accommodation. The explanation of this curious symptom is probably the vertical diplopia which frequently recurs. As the angle of separation is so small, the images of the printed letters overlap when close to the eyes and hence cause great confusion; whereas at a distance the vertical separation will be greater, so that less inconvenience is occasioned, or one image may be suppressed.

In some cases hyperphoria is *latent* and cannot be revealed by the ordinary tests. Whenever from the symptoms one has reason to suspect that this is the case, it is well to cover up one eye completely for 24 hours and then test the patient with the Maddox rods immediately after uncovering the eye.

It is, however, most important to remember that heterophoria may exist to a marked degree without giving rise to any symptoms whatever; in such cases it is unnecessary to order prisms, or to subject the patient to any of the various operations that have been devised for its relief. The symptoms of the affection are, however, so protean, that the cause is much more likely to be overlooked than to be overtreated.

When applying the tests for hyperphoria it is most important to repeat them with the head of the patient in different positions, as by this means the *comitant* cases can be distinguished from the *noncomitant* cases. It is well known that in a right convergent comitant strabismus (R.C.C.S.) the deviation of the right eye is at any rate

approximately the same whether the eyes are ranging towards the right or towards the left. Late changes in the texture of the right internal rectus may interfere with the absolute constancy of the deviation, but practically the deviation remains nearly constant in any ranging movement. It is very different in paralytic or paretic cases. If there be a parĕsis (πάρεσις) of the left sixth nerve and the attention of the patient be directed towards the left, the right eye will turn to the left, while the left eye will only rotate slightly in that direction. In other words, an esotropia will be observed on ranging to the left, but not on ranging to the right ; the deviation will be *non-comitant*.

All noncomitant heterophorias and heterotropias are due to nuclear or subnuclear lesions. All comitant heterophorias or heterotropias are due to some lesion or disturbance above the nucleus. As the deviation of a prism is practically the same in whatever direction the eye behind it ranges, the treatment by prisms is always satisfactory for the relief of *comitant* hyperphorias. For recent *noncomitant* hyperphorias and paralytic squints I invariably give iodine in this mixture,

℞	Tinct. Iodi Fort.	ʒss
	Pot. Iod.	gr. x
	Acidi Citrici	ʒij
	Aq.	ad ℥x

℥ss. t.d.s. ex aq. semihora ante cibum.

This causes no depression, no rash, is as effective as gr. xij–xv of the usual potassium iodide, and is far more pleasant to take.

I. STATIC TESTS.

These tests determine the position of the eyes when at rest.

I begin with the old screen test which I practically never use now, as I get much more accurate results with the subjective test with Maddox rods ; the objective test for heterotropia that I prefer is Priestley Smith's method

by corneal images (*p.* 100), but I cannot detect very minute errors in this way with any confidence.

1. Screen Tests. (*a*) *Objective.*—The test object should be a small electric light on a black background; but practically the light of the tangent scales (*p.* 101) or even a small white spot in the centre of a blackened target of cardboard will answer quite satisfactorily. The patient is placed 6 metres distant from this target, with his head erect and his eyes looking straight forwards at the test object. The surgeon now suddenly covers the right eye with a card, and notices whether this right eye moves in any direction when covered.

If the right eye *alone* moves when covered, while the left eye remains fixed on the test object, *heterophoria* is present. If the right eye moves outwards, exophoria; if inwards, esophoria; if upwards, right hyperphoria; if downwards, left hyperphoria.

If *both* eyes move when the right eye is covered, the *left* eye is squinting, i.e., *heterotropia* is present.

If *neither* eye moves, either orthophoria is present or the right eye is squinting; in the latter case, the deviation is easily seen on covering the left eye. If orthophoria be present, on repeating the test with a prism of 4 ▽ held before the left eye, first base out, and then base in, it will be found that the covered eye deviates an *equal* amount, first outwards and then inwards, owing to the 4 ▽ apparent displacement of the light by the prism.

Now, after holding the card over the right eye for at least a minute, suddenly remove it, when a corrective movement in the opposite direction will be seen. This movement of "redress" is usually much more easily seen than the movement of deviation just described.

Duane has added a refinement to this test by holding prisms of different strengths before the right eye in the appropriate positions until the movement is unobserved. This will probably be found to be the case with two or

three prisms, e.g., 4 ▽, 5 ▽, and 6 ▽, then the heterophoria is probably of 5 ▽.

Test for *comitancy* by repeating the test with the patient's head in different positions. Thus, if right hyperphoria be present, repeat the test with the patient's head inclined downwards, and again when tilted upwards. If the deviation is the same in each of these positions, simple (or comitant) hyperphoria is present; if it is much more marked in one position than another, paralytic hyperphoria is diagnosed. For instance, if the deviation is most marked when the patient's face is directed upwards and to the right, his eyes must be directed downwards and to the left; as this is the direction in which the greatest deviation occurs, there must be some weakness of the right superior oblique (or of the left inferior rectus if left hyperphoria be present. *See pp.* 106–7).

When oblique deviations occur, decompose them into their vertical and horizontal components. Correct the vertical component first, and then study the horizontal component.

b. Subjective.—Apparent movement of the object, sometimes conveniently, although somewhat loosely, called *parallax.*

When the right eye behind the screen is deviating inwards, on suddenly transferring the screen to the left eye, the object will appear to make a jump to the right. This is called by Duane *homonymous parallax,* as it is analogous to *homonymous diplopia,* for instead of the double images being seen simultaneously they are seen in succession. Its amount may be tested in the same way with prisms, by finding the strength of prism that will abolish this apparent movement. This test I personally consider unreliable, for in the first place it requires an intelligent patient for its perception, and as the movement of redress is in the opposite direction to that of the deviation, it is sometimes this movement of

redress which is noticed, and not the primary jump, so that confusion may easily result. I may say that Duane has himself pointed out these sources of error.

2. **Corneal Images.** *Objective.*—Priestley Smith first devised this method to be used with his tangent scale for the determination of the angle of squint, and now Maddox is the ablest advocate of this exceedingly useful method of detecting slight deviations; the reader is referred to Maddox's book on *The Ocular Muscles* for a fuller account of the procedure. The patient is directed to look at the central hole in the mirror of the ophthalmoscope, with which the surgeon throws the light first on one cornea and then on the other. The corneal reflection of the illuminated mirror appears as a small bright spot of light which "maps out with sufficient precision for clinical purposes the point in each cornea which is traversed by the visual line." It is important (*a*) To keep the patient's attention fixed on the mirror, as then the visual line of the patient coincides with that of the surgeon; (*b*) To judge of the position of the bright spot with reference to the cornea and not to the pupil, which is usually eccentric.

In normal eyes these corneal reflections occupy *symmetrical* positions in the two corneæ, nearly always slightly to the inner side, for a reason that will be given later (owing to the angle β, *p.* 116). If on covering each eye successively there is no movement of the corneal image on the other eye, one may be sure that there is no deviation, even though there may be slight asymmetry owing to a difference of β in the two eyes.

If there is *asymmetry*, and movement on covering, heterotropia is present. In this way the distinction between real and apparent squint can be at once made.

The test for *comitancy* is made by turning the patient's head in different directions and noting, while he is still fixing the mirror, whether the position of the corneal image in the squinting eye is unchanged; if there is an

alteration in its position, the squint is noncomitant or paralytic.

3. **Maddox Rods**.—The mounted row of red glass rods and the tangent scales that were introduced by Maddox are too well known to need any detailed description. The rods must be accurately cylindrical; if they taper in the least, they are useless, as if placed vertically a hyperphoria will be suggested which does not exist. Such faulty instruments can be easily recognized by the fact that the deviation of the red line upwards will be converted into a deviation downwards on turning the rods upside down.

It is the simplest, most rapid, and most reliable of any subjective test for detecting any want of balance in the ocular muscles. The tangent scales should be fixed to the wall at the end of the consulting-room in the form of a cross coloured a dull black, one limb being horizontal, the other vertical, with a small electric light at the zero point of intersection of these scales. The scales above and to the right carry white figures denoting centrads, while those below and to the left carry blue figures.

a. Vertical.—The determination of the hyperphoria present must be minutely accurate, so to avoid any error due to a faulty centring of the lenses I make this test whenever possible without providing the patient with his refractive correction.

The rods are placed vertically in the right cell of the trial frames, and a green glass is placed in the hinder cell on the left. The patient, armed with these trial frames at 6 metres' distance, is directed to look attentively at the light. His right eye will see a red horizontal line, while his left eye will see the light coloured green. The eyes, being thus dissociated, will take up a position of rest, and if the red line passes exactly through the light, there is no hyperphoria.

If the right eye has a tendency to deviate upwards, it will be revealed by the displacement downwards of the red

line below the level of the light. The patient can at once say through which blue figure on the vertical scale the red line passes. This tells us at once the amount of the hyperphoria in centrads.

Should the patient have such deficient visual acuity that he cannot distinguish the figures on the scale, we place a 5 ▽ prism exactly horizontally in the anterior cell of the trial frames before the left eye. The patient will tell us that the red line occupies the same place, although the light has moved horizontally, but on slowly rotating the prism base upwards we shall find one position in which the red line passes through the light. On now reading off the angle at which the edge of the prism is pointing, we can at once determine the extent of the hyperphoria. Suppose, for instance, that the edge of the prism points to 30°, we know from *p.* 90 that $\phi = \delta \sin 30°$, and on referring to *Table III*, *p.* 235, we see that $\sin 30° = \cdot 5$,

$\therefore$ the right eye tends to deviate upwards 2·5 ▽.

If Maddox's prism verger is at hand, even this little trouble is avoided; we have only to turn the milled head until the red line passes through the light, and then the prism used before each eye can be directly read off from the index on the right celluloid scale.

The prism verger consists of a frame in which two prisms, each of 6° d., 6 ▽ or 12 ▽, are so mounted as to be simultaneously rotated in opposite directions by turning a milled head.* With the 12 ▽ verger that I use, and to which I shall always refer in this book, the reading in the above case would be "R eye up 2½ ▽."

I personally always test for hyperphoria first, and find that much time is saved by rotating a strong prism in this way instead of holding up different prisms until one is

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* Maddox's prism verger may be obtained from C. W. Dixey & Son, 3, New Bond Street, London, W. The prisms required should be specified; I recommend 12 ▽ prisms.

found that corrects the defect. The statements of the patient can always be checked by noting whether they are consistent with the prism used to correct or to over-correct the displacement.

The test for comitancy is very easily made by the patient tilting his face downwards and then upwards, and noting if the deviation of the red line remains the same. It will be probably necessary to remove the trial frames and to hold the Maddox rods in the hand when this test of comitancy is made.

b. Horizontal.—Before testing horizontal deviations it is essential that the patient should always wear the lenses that correct his refractive errors, and any prism that may be required to correct his hyperphoria.

The glass rods are now placed horizontally in the trial frames, when the patient will see a vertical red line with his right eye, and he is asked through which figure on the horizontal scale this red line passes. This gives the amount of deviation: if it is to the right (the white figures) the case is one of esophoria, if to the left (the blue figures) of exophoria. If the red line is continually shifting its position, uncorrected anisometropia or paresis is probably present. This dancing of the red line is much more frequently complained of when testing horizontal deviations than vertical deviations, and can be used with advantage in detecting any slight anisometropia that may remain still uncorrected. Even as small an error as ·25 D is often revealed by this method; when the anisometropia is accurately corrected, the red line remains stationary.

When the patient cannot read the figures, a similar procedure with a rotating prism or the verger must be undertaken. The prism may now be stronger, say 10 ▽, and in this case must be set vertically, and rotated until the red line passes through the light; then, on reading off the angle at which the edge points, and remembering

that $\theta = \delta \cos \rho$ and referring to *Table III, p.* 235 we find the amount of the horizontal defect.

The Maddox Wing Test for Heterophoria.—This is an extremely ingenious test for every kind of heterophoria at a distance of $\frac{1}{3}$ m. I strongly recommend the "Hospital Pattern"; it will never get out of order, whereas in my hands the more elaborate pattern was continually requiring readjustment. An arrangement of screens prevents the right eye from seeing anything but the white hand and the red arrow, while the left eye sees only the chart. The red arrow points to a vertical row of figures and records any vertical deviation, while the white hand points to a horizontal row of white figures and records any horizontal deviation. It should be remembered that there is usually some exophoria at a distance of even $\frac{1}{3}$ m. of 4° to 6° (say 7 ▽ to $10\frac{1}{2}$ ▽) which may be regarded as physiological, as it gives rise to no symptoms and should not be corrected. Esophoria for near vision should always be corrected; it will be found that when convex lenses are required it will often be sufficient to increase their power and to indicate on the prescription form that the reading glasses should be centred as for distance.

The instrument also gives a method of determining any torsion of the eyeballs that may occur when reading at this distance. Further reference to such torsional errors will be found on *p.* 136.

II. DYNAMIC TESTS.

These tests determine the ability of the eyes to move. We must first distinguish between a noncomitant paralysis of one or more muscles and a comitant loss of an "associated movement." Thus a loss of convergence is easily distinguished by the complete freedom of the ranging movements from a paralysis of both internal recti. We will first briefly consider the paralysis of one or more

ocular muscles, although it has less to do with the purpose of this book than the anomalies of the associated movements.

Ocular Paralyses.—The symptoms are :—

1. *Limitation of movement* of the affected eye in some cardinal direction. The cardinal directions are up, down, in, and out. As the eyes move together in one of these directions, one eye lags more and more behind the other, producing a continually increasing deviation. This gives rise to

2. *Diplopia*, which is most marked in the cardinal direction of the action of the affected muscle ; with the eye in a given position the false image is displaced as the muscle, if healthy, would displace the eye.

Maddox has pointed out the inaccuracy of the aphorism, that the affected muscle is the one which should turn the eye in the direction of greatest diplopia, unless the word *cardinal* is prefixed to direction. If the eye be turned outwards, the superior rectus raises it most strongly ; if turned inwards, the action of the muscle is mainly intorsion.

3. *Altered Position of the Head.* The face looks in the direction of the greatest diplopia, so that the eyes may be directed towards the single vision area. In paralysis of the right superior rectus, the diplopia is greatest in the right superior area ; hence, if the patient directs his face upwards and to the right, he will be least inconvenienced by his diplopia, as for an object immediately in front of him he will have to direct his eyes downwards and to the left.

The diplopia gives us the key by which we may solve all these problems. We must first consider the line of action of the muscles. The external and internal recti of course act in the horizontal direction ; the superior or inferior rectus elevates or depresses the eye simply (i.e., without any torsion) when it is turned 27° outwards ; the inferior or superior oblique would simply elevate or depress the eye if it were turned 51° inwards.

Hence, both eyes will be turned to the right by the action of the right external rectus and the left internal rectus; when they are in this "eyes right" position, both eyes will be elevated by the action of the right superior rectus and the left inferior oblique almost entirely.

Consequently, the twelve muscles of the eyes may be divided into three groups of four each, four moving the eyes laterally, four upwards, and four downwards. Each group is divided into two pairs, one muscle of each pair being in the right eye and one in the left eye.

1. Lateral	*a.* Right turners	R. ext. rect. and L. int. rect.
	b. Left turners	L. ext. rect. and R. int. rect.
2. Elevators	*a.* Eyes right	R. sup. rect. and L. inf. oblique
	b. Eyes left	L. sup. rect. and R inf. oblique
3. Depressors	*a.* Eyes right	R. inf. rect. and L. sup. oblique
	b. Eyes left	L. inf. rect. and R. sup. oblique

Each of these six pairs of muscles consists of Graefe's "True Associates," and are quite easily remembered by Maddox's mnemonic that their names are the most contrary possible, e.g., right external rectus and left internal rectus; or left superior rectus and right inferior oblique.

It is clear then that if we consider the patient's field of view to be divided into *right* superior, external, and inferior

Left Superior		Right Superior
Left External		Right External
Left Inferior		Right Inferior

areas, and *left* superior, external, and inferior areas, and if we find the area in which the greatest diplopia occurs, the

condition must be due to paralysis of the "same-named muscle or the most cross-named muscle." As the diplopia is only considered in the cardinal directions, any source of error from a pre-existing heterophoria is eliminated.

Thus, if the maximum vertical diplopia occurs in the right superior area of the patient's field, he must have a paralysis of the same-named muscle, i.e., the right superior rectus, or of the most cross-named muscle, i.e., the left inferior oblique.

It only remains to distinguish which eye is affected. By screening one eye, while the object is held in the area of maximum diplopia, the corresponding image disappears. "The image that lies farthest in the direction of increasing diplopia belongs to the paralysed eye."

If the diplopia is increased in more than one of these directions, an affection of more than one muscle is indicated.

Left sixth paralysis will give a maximum lateral diplopia in the left external area, but careful examination will usually show that the lateral separation of the images will be slightly greater in the left inferior area than in the left superior area. This is not in accordance with the simple rule given above, but it is no doubt due to the fact that the converging centre is practically always active when we are looking downwards and not when we are looking upwards, and consequently the convergence will be greater in the left inferior area.

But there are other minute points that may be observed that are rather puzzling; in the left inferior area the false image will be slightly intorted and slightly higher than the true image, while in the left superior area it will be slightly extorted and lower than the true image. By extorsion I mean a rotation such that the upper extremity of the false image is turned outwards. I regret that I am forced to differ from Maddox in his explanation of this point. He attributes it to the fact that the oblique muscles will gain in vertical effect what they will lose in torsional effect, owing to the median position of the paralysed eye. But unfortunately when the eyes are in the "eyes left" position, neither of the left oblique muscles is called into play when the left eye attempts to direct itself to the left superior or the left inferior area. The explanation is, I think, far simpler than that. What is the action of the external rectus when the eye is raised obliquely to the left

superior area ? Clearly its insertion is raised, and so the effect of its contraction must at least be to turn the eye outwards and slightly downwards. The axis of rotation (i.e., the line drawn through the centre of motility parallel to that at right angles to the direction of the muscle's action) must be tilted so that its upper extremity inclines slightly backwards. But a little consideration will show that there must be another effect. When the left external rectus contracts with the cornea facing the left superior area, the cornea must be drawn outwards, slightly downwards, and a trifle extorted. When the cornea faces the left inferior area the left external rectus will abduct it and also slightly raise it and intort it. But we know that the false image is displaced as the healthy muscle would displace the eye, so these minute points are fully explained by the action of the external rectus when the eye is not in the primary position.

It is well to make this quite clear, as otherwise one might be inclined to suppose that that there were present also some paresis of an elevator and a depressor. Similar observations may be made when there is a complete paralysis of the internal rectus alone, but I have never met with such a case.

It will be noticed that no attention is paid to the torsion of the image, or whether the diplopia is homonymous or heteronymous, in the diagnosis of an ocular paralysis by this method ; these are refinements which may only lead to trouble. For instance, in fourth-nerve paralysis the diplopia is usually homonymous, but it may be heteronymous if any exophoria be present, and the torsion may not be what one would expect owing to an unrecognized cyclophoria, or periphoria, as I prefer to call it. It will be found that by this method all difficulties disappear. The only difficulty that may arise in making this test is when the paretic eye is the " master eye " and is used for fixing ; a strong secondary deviation of the other eye will then be made, which at once reveals the condition. A few years ago I had a case of left sixth-nerve paresis ; there was marked lateral diplopia in the left external area, at the same time the right eye was seen to turn very strongly inwards. On covering the left eye the left image disappeared, but the right eye moved outwards,

showing that it was not fixing the object. An intelligent student at the time diagnosed the case wrongly as spasm of the right internal rectus. I subsequently found a minute old macular hæmorrhage in the right eye.

If both eyes can see well, an ocular paresis however slight can be discovered with the Maddox rods before one eye and turning the head in the required position so that the displacement of the red line can be investigated in each of the named areas.

Spasm.—The overaction of one muscle, say the right external rectus, would, on a cursory examination, resemble a paralysis of its opponent, the right internal rectus, or possibly of the left external rectus; but the distinction would be easily made. A spasm of the right internal rectus would manifest itself most vividly when that muscle is called into action, i.e., both on ranging to the left and on convergence. Its characters, moreover, would be a *sudden* and *temporary* deviation of one eye, and absence of any marked heterophoria (revealed by Maddox's rods) in the intervals between the attacks. Normal movements in the other eye of course exclude paralysis of the left external rectus. When a sudden temporary deviation of one eye occurs, it is nearly always due to a heterophoria, or latent squint, which becomes manifest when the nervous centres are exhausted. I believe I have only seen one case of this very rare condition, a spasm of the right external rectus occurring during a meningitis, afterwards followed by a paralysis of the right sixth.

Secondary Actions of the Individual Muscles.—If the eye be originally in the primary position, certain secondary actions of some of the muscles should be noted, as they will explain for instance the usual homonymous diplopia of fourth-nerve paralysis.

1. The superior and inferior recti, as they are inserted anteriorly, slightly *adduct* the cornea, while the obliques, being inserted posteriorly, slightly *abduct* the cornea.

2. The superior muscles (superior rectus and superior oblique), as they are inserted superiorly, *intort* the cornea; while the inferior muscles *extort* the cornea.

All these points depend upon the fact that *all* the ocular muscles arise from origins nearer the median plane than their insertions. For a fuller and more detailed account of the ocular movements and their paralyses I would refer the reader to Maddox's *Ocular Muscles*.

Associated Movements.—Those in the horizontal direction are somewhat difficult, and we shall have to devote some time to their study. The associated movements in the vertical direction, supra- and infra-vergence, are simpler, and therefore we shall describe the tests for these movements first. These tests determine what has been called "the breadth of fusion," i.e., the strength of the prism, base up and base down, that can be overcome by the eyes. For this purpose Dr. Maddox's verger is invaluable, as it enables us to carry out the tests much more quickly and more accurately than was possible before.

Vertical Deviations. *Supravergence.*—Adjust the prisms in the verger so that the edges of both point to the patient's left, and, placing the instrument on his face, direct his attention to the test object previously described, i.e., a small electric light or a white spot in the centre of a blackened target. On now rotating the milled head clockwise until two white spots are seen by the patient, and noting the reading of the instrument, we find the limit of the supravergence of the left eye; on then rotating the milled head counter-clockwise we similarly find the limit of supravergence of the right eye. With the 12 ▽ verger the reading is usually about 1 ▽ or 2 ▽ for each eye. The actual breadth of fusion is in this case unimportant; the important point is the difference between the supraverging power of the eyes. In true orthophoria, the right eye will be able to move up above the level of its fellow to exactly the same extent as the left eye will be able to move up.

Should there be a difference, half the difference determines the amount of correction that may be given to the eye. In other words, the difference between the readings on the instrument gives double the total hyperphoria. This test is very useful in those cases in which the rod test is unsatisfactory owing to the dancing of the line of light.

Thus suppose that the reading, when the rotation was counter-clockwise, was R eye up 3 ▽, and when the rotation was clockwise L eye up 1 ▽, there must be $\frac{1}{2}(3 - 1)$ or 1▽ of right hyperphoria present.

Treatment of Hyperphoria.—If hyperphoria is found, and there are no symptoms whatever attributable to it, do not correct it. If there are symptoms, it is important to distinguish between comitant hyperphoria and paretic hyperphoria.

In all cases of recent *paretic* hyperphoria try what medicinal treatment will do. Personally I always give the iodine mixture mentioned on *p.* 97; electrical treatment of the paretic muscle is said to be occasionally of service. I have no faith whatever in exercising prisms in non-comitant cases, they only act on associated movements, not on a single paretic muscle. I do not now hesitate to order in chronic cases even the full correction if that is necessary to relieve the symptoms. For years I feared that patients would require stronger and stronger relieving prisms, but I have found that it is far more common to come again because they need weaker ones. In all cases of hyperphoria trials at reading distance with the Maddox wing test should also be made *with the required reading glasses*, as different prisms may be required for this purpose. In no paretic case can complete relief be given by relieving prisms, as in such a case the amount of the defect will vary in different parts of the field; but on moving his head rather than his eyes the patient will probably get relief from all his urgent symptoms.

Comitant hyperphoria is easily recognized by the fact

that the deviation remains the same whether the head be tilted forwards or backwards, and in such cases the proper use of exercising prisms may entirely cure the condition, though I personally have not had so much success as is claimed by others. If, for instance, the right eye tends 1 ▽ upwards, let the patient wear a prism, say of 1 ▽ base upwards, before the right eye for five minutes at a time when he is not fatigued, say after meals. This will exercise his verging power ; as soon as he tends to acquire a vertical diplopia, the prism should be removed, and he should close his eyes for a few moments.

Maddox has suggested the use of circular prisms mounted in special frames, so that the position of the prisms in the frames can be altered, if necessary, at each visit to the surgeon. When the edges of the prisms are both directed towards the left, they will have no vertical effect on the eyes ; but by rotating one prism a trifle upwards and the other a trifle downwards, the required amount of exercise can be given.

Relieving Prisms.—If the patient does not wish to be bothered with these tedious exercises, there is no occasion to insist on them, as hyperphoria is usually very slight in amount, and as relieving prisms are invariably satisfactory. I now correct the whole defect found by the distance test, if that is the most comfortable to the patient, and, as said above, I find that the hyperphoria does not tend to increase.

From the note on *p.* 91 it will be seen that if reliance were placed on the near test, an over-correction would be given for distance. In relieving prisms, the edge of the prism must point in the direction of the deviation ; thus, for right hyperphoria the right prism must point edge upwards and the left edge downwards. When a refractive error needs correction, it is always more convenient to order the glasses to be decentred to the amount required to give the prismatic relief by the formula $l = \frac{10\ N}{D}$

where N denotes the number of centrads required for the relief of the hyperphoria, and l the amount of decentration in millimetres. It is customary to denote a prism by the position of its base; consequently the equivalent of a + 4 D lens combined with a 1 ▽ prism base up, is given by decentring the lens l mm. up,

$$l = \frac{10}{4} = 2{\cdot}5 \text{ mm.}$$

The lens must therefore be decentred upwards 2·5 mm., the optical centre being 2·5 mm. above the geometrical centre of the lens. If the prism were required base down, the decentration should be downwards. In fact whenever the result does not carry the negative sign, decentre in the same direction as the base of the prism should be.

If, however, it were required to give a 2·5 ▽ prism base up before the right eye of a − 5 D myope.

$$l = \frac{25}{-5} = -5 \text{ mm.}$$

The minus sign reminds us that the decentration must be in the opposite direction to that of the base of the prism, i.e., in the direction of the edge of the prism, so in this case the decentration must be downwards. It would be better to divide the correction between the two lenses and order R − 5 D decentred down 2·5 mm., L − 5 D decentred up 2·5 mm. Before definitely ordering this or any other prismatic correction, it is advisable to let the patient wear the prisms in the waiting-room for at least twenty minutes in order to be sure that they relieve the discomfort from which he suffers.

Points to remember about Hyperphoria.—

1. Test hyperphoria for distance without the correcting glasses if possible.

2. Never correct hyperphoria unless symptoms of the defect are discoverable.

8

3. Small errors such as 1 ∇ may cause great discomfort, although high errors, even up to 6 ∇ or 7 ∇, may cause no inconvenience.

4. Treat hyperphoria by relieving prisms, the edges of the prisms in the direction of the deviation.

If *comitant*, it is well to persuade the patient to endeavour to cure the condition by the use of exercising prisms. In recent *paretic* cases give iodine.

5. Let the patient wear the correction for twenty minutes before finally ordering it. Possibly a slight alteration may be required in the prism, as a concave lens with a prism always diminishes its effect, while a convex lens usually increases it (*p.* 215). Also, if necessary, a stronger prism may be given for close work in comitant cases for the reason given on *p.* 91, and in noncomitant paresis of an adductor or a depressor for the additional reason that the action of these muscles is then especially demanded.

Horizontal Deviations. — Before dealing with this intricate subject, we must define certain lines and angles that are of importance in understanding some conditions, and the literature dealing with them.

The Visual Line and the Fixation Line. — In *Fig.* 9 the line AK′K″ϕ″ represents the optic axis, F the fovea, and O the object viewed. What is called the *visual line* consists, strictly speaking, of two parallel lines, one OK′ drawn from O to the first nodal point, the other K″ F drawn from the second nodal point to the fovea. It will be noticed that the visual line cuts the cornea on the inner side of the optic axis in the diagram, as F lies to the outer side of ϕ″. This is always the case in emmetropia and hypermetropia. Thus, if a hypermetrope view some distant object so that his visual lines are parallel, his optic axes will diverge. This divergence is less in myopia; in extreme degrees of myopia the optic axes may even converge.

It is found that slight movements of the eyeball may be considered as movements of rotation about a fixed centre M, called the centre of motility, which is usually about 13·4 mm. behind the cornea, i.e., about 27 mm. (denoted by k) behind the plane of the spectacles.

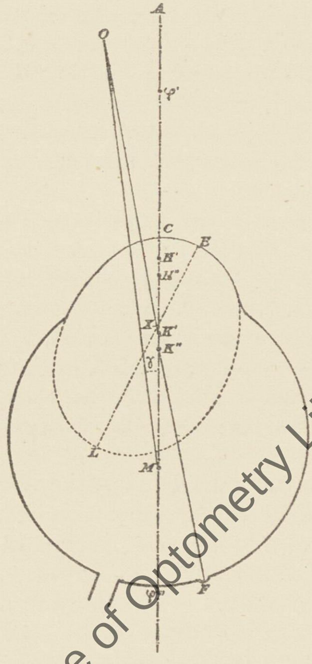


Fig. 9. (After Landolt.)

The *fixation line* is the line MO joining the centre of motility and the object viewed.

The angles α, β, γ, *and* κ.—The angle α is now used in two different senses. The angle α of Landolt is the term used to denote the angle OXE between the visual line and EL, the major axis of the corneal ellipsoid. We now know

from the investigations of Tscherning and Sulzer that the shape of the cornea is not that of an ellipsoid of revolution and that it has no axis of symmetry.

The angle α of Donders was the angle between the axis of the cornea and the visual line. Now, as Donders assumed wrongly that the axis of the cornea coincided with the optic axis, it is clear that his angle α is AK′ O.

In order to avoid this confusion, the angle AK′O is called by Brubaker the angle β. I do not know whether he was the originator of this term, but it seems to me the best to use, as it has no ambiguous meaning.

The angle γ is the angle AMO between the fixation line and the optic axis.

Landolt's angle κ is the angle that the fixation line makes with that normal to the cornea that passes through the pupillary centre. This angle is a concession to hurried workers, as the centre of the pupil is much easier to see than the point where the optic axis cuts the cornea. It is not, however, to be recommended, as the pupil is always eccentric, being usually placed to the inner side, and it is not uncommon to find that the pupillary opening is asymmetrical in the two eyes.

All these angles are considered positive when the visual and fixation lines lie to the inner side of the optic axis, which is nearly always the case except in high myopia. The variation of these angles in different cases depends of course on the situation of the fovea (F).

Apparent Squint.—It is obvious that, with a high angle β, there will be an appearance of external squint, for the corneal centre C will be displaced outwards. The diagnosis is at once made by examining the corneal images with the ophthalmoscope; when these images are symmetrically placed, while the patient is gazing at the mirror, there is no real squint.

As we have already described the static and dynamic

tests of the external and internal recti, it only remains to consider the dynamic tests of the associated movements of convergence and divergence. As I have already said, it is most important first to correct all refractive errors and any hyperphoria that may be present, before investigating the function of convergence.

Since especial precautions must be taken that no experimental error is introduced by any displacement of the centres of the trial glasses, it is well to consider what means we have to avoid this error. We will first find how we can so adjust the trial lenses that their optical centres are exactly traversed by the visual lines of the patient.

Centring the Glasses.—It is usually only considered necessary to see that the centres of the correcting glasses are opposite the centres of the pupils; but this is not accurate work. The centres of the glasses must be traversed by the visual lines of the patient, and, owing to the variability of the angle β, the visual lines may pass by the extreme inner margin of the pupil, or they may not pass through the pupil at all. It is here that the method of corneal images renders us such invaluable service when a Maddox localizer is used. The localizer with its two sights is placed before one eye in the trial frames, and the surgeon alters the width of the frame until the corneal reflection is seen to be intersected by the sights of the localizer. The distance is read off in millimetres, and then the other eye is tested in the same way. It will be found that this measurement is an exceedingly troublesome one to make, and fortunately it is quite unnecessary before testing the convergence of the eyes. It is true that, if we needed to know the actual deviation of the fixation lines, the utmost accuracy in centring the glasses would be required, as an error of half a millimetre in each eye when $-$ 10 D are worn would entail a total error of convergence of 1 ∇. For the purpose, however, of ordering relieving glasses, we need

only know what will give the requisite relief, without investigating the actual effect which the prescribed glasses have on the ocular fixation lines.

It is important to realize that the prisms that produce a given deviation on the uncorrected eye, will produce a different effect when correcting glasses are worn (*p.* 215). The tables there given I have calculated to show the difference that convex and concave glasses make on this deviation, but in practice they will rarely be found useful if the simple method that I recommend be adopted.

Place the correcting glasses in a trial frame, of which the distance between the centres is known ; suppose that this distance is 60 mm., quite irrespective of the actual distance between the visual lines of the patient, and test the patient's convergence. Allow me to anticipate the results of this test—say, finally he is found to require abducting prisms of 1 ▽ before each eye ; the distance between the centres of the glasses can be immediately written down according to the formula $l = \frac{10\,N}{D}$. In this case the prism is abducting, *base in.* Suppose that the patient required R + 5 D, L + 4 D, and was wearing these in the trial frames. Clearly, the right glass must be decentred $\frac{10}{5}$ or 2 mm., i.e., 2 mm. inwards, while the left glass must be decentred $\frac{10}{4}$ or 2·5 mm., i.e., 2·5 mm. inwards. As the centre of each lens in the trial frames was 30 mm. from the mid-nasal line, the optical centre of the right glass in the spectacles must be 30 — 2 or 28 mm., while that of the left glass must be 30 — 2·5 or 27·5 mm. from the centre of the nose. It does not matter in the least what the distance between his visual lines is, this is his correction, and the prescription should be written

R. + 5 D optic centre 28 mm. from median line.
L. + 4 D ,, ,, 27·5 mm. ,, ,,

Opticians always measure the interpupillary distance, and it is well, for cosmetic reasons, that the geometrical centres of the glasses should be opposite the pupils ; and any competent optician can be trusted to carry out such a prescription perfectly, whatever may be the distance between the patient's eyes.

For instance, in the above case, if the patient's right eye were 32 mm. from the median line, while his left eye were 31 mm. from his median line, the right lens should be decentred 32 – 28 or 4 mm. inwards, and the left lens decentred 31 – 27·5 mm. or 3·5 mm. inwards. The essential point is that the optic centre of each lens should be in the correct position as prescribed, and there is no necessity for the surgeon accurately to measure the ocular distance of each eye from the median line.

When the anisometropia is greater than + 1 D, or if for any other reason the decentration is greater than can be managed (say 8 mm.), it will be the duty of the optician to supply an equivalent prismosphere, i.e., the spherical surface ground on the appropriate prism. In exceptional circumstances, if the optician is reliable, he may be empowered by the surgeon to decentre the weaker lens *ad maximum* and the stronger to a suitable extent, and then to *displace* both lenses so that their optic centres are in the required position.

CONVERGENCE.

We will suppose then that our patient has had his hyperphoria and his refractive errors corrected, and we now proceed to test his range of convergence (R_c) for distance and for close work ; these are most important tests, as will be seen presently.

Minimum Convergence or Divergence.—The patient is directed to fix a small light, or one of the smaller letters of the test types at 6 metres' distance, while the surgeon with different prisms (held edge out) discovers the strongest abducting prisms that the patient can overcome. As soon as this limit is passed, he will see two objects. For this purpose the verger is most convenient, as with it the

test can be very expeditiously made, which is of importance, as all these tests are very tiring to the patient. Normally about 3 ∇ or 4 ∇ base *in* before each eye can be overcome with single vision. Let us suppose that a divergent prism of 3 ∇ can be overcome by each eye; the result will be read on the verger—Divergence 6 ∇, and should be noted as Total Divergence or TD $= -6\nabla$.

Relative Range of Convergence or Breadth of Fusion at 6 metres.—By rotating the milled head of the verger in the clockwise direction, a series of adducting prisms are practically placed before the eye, and in this way we find what is the maximum convergence that the eyes can exercise when the accommodation is relaxed. It is necessary, in executing this test, that the patient should fix his attention on the smallest letter that he can see at 6 metres' distance, and warn us as soon as the letter becomes indistinct, or becomes double. Suppose it is found, when great care is taken that the small letters can still be distinctly seen, that 6 ∇ base out before each eye can be overcome; if the patient is allowed to exercise his accommodation, a very much greater amount of convergence is possible, but this will be discovered by his failing to read the small type. The reading on the verger, which gives the total convergence, will be 12 ∇ and should be noted as TC $= 12\nabla$; so the total range of convergence will be

$$\text{TC} - \text{TD or } 12 + 6 = 18\nabla$$

Now as the final correction is to be divided equally between the two eyes, it is simpler to note the range of convergence R_c of each eye, so $R_c = 9\nabla$ for each eye. The range is denoted by a length and carries no sign of direction, but the point from which the range is measured -3∇ of divergence for each eye must be marked with the negative sign, while the positive sign indicates convergence. The note does not mean that the patient's fixation lines diverge and converge to exactly this extent; the

actual amount they can move depends on the glasses that he is also wearing ; the note merely implies that he can overcome these prisms when wearing his correcting glasses 60 mm. apart, and that practically is all we need know.

In these cases as there is such a wide range of convergence possible at each distance viewed, there must be " a region of comfort," and so I make no use of the Maddox rods for convergence defects except for hurried hospital work, and as a rough indication whether further examination is required. I have found that practically every patient can continuously exercise only the middle third of his relative range without fatigue.

$$-3\,\triangledown \qquad\qquad 0 \qquad\qquad +3\,\triangledown \qquad\qquad +6\,\triangledown$$

In this case he will be able to exercise the range from 0 to + 3 with comfort, so he will not require any relieving prisms for distance. The Maddox rods would indicate an esophoria of about 2 ▽ or 1 ▽ for each eye.

Now, most hypermetropic patients have a considerable power of adapting themselves to the new conditions caused by a correction of their refractive errors, so that even if R_c for distance were -1 to $+5$ centrads, I should not give the indicated correction of + 1 ▽ centrad for each eye, if he were a hypermetrope. Most probably, after a day or two of discomfort, he would show in three weeks a much greater power of overcoming divergence.

It is simplest to draw a diagram as above, but of course there is no necessity to do so. When there is esophoria the prism indicated is given by adding (algebraically) the minimum to one third of R_c. Thus in the case above

$$\text{Min.} + \tfrac{1}{3}R_c = -1 + \tfrac{1}{3}6 = -1 + 2 = +1\,\triangledown$$

so that a prism of 1 ▽ base out is indicated for each eye in order to give relief from discomfort *at once.*

Relative Range of Convergence at Reading Distance.—I find my simple little dynamometer (*p.* 15) will serve

admirably for this test. It is held at the ordinary reading distance from the patient's spectacle plane, 30 cm., or whatever the distance may be. With the verger his minimum convergence is found ; we will suppose that he can just maintain single vision of the slit, and define the engraved line when the verger indicates — 2 ▽ of divergence. By insisting on the patient's saying the moment that the fine line appears blurred, we have some guarantee that no more than the due amount of his accommodation is exercised, we are supposing, of course, that his reading correction, if any, is being worn. On now rotating the milled head of the verger in the clockwise direction we find the maximum adducting prism he can overcome. In one case I found that the patient could overcome an adducting prism of + 10 ▽ before each eye. It is seen that her R_c was excellent, being 12 ▽ ; but it was in the wrong place.

— 2 R + 2 ▽ + 6 ▽ + 10 ▽

Here R denotes the convergence required for reading, 2 △ to the left of the area of comfort, so that 2 ▽ prisms base out should be the smallest correction that would relieve her.

Or Min. $+ \frac{1}{3} R_c = -2 + \frac{1}{3}(12) = +2$ ▽

I may state that this patient had consulted many experts all over the country, and though her refractive errors had been accurately corrected by most of them, they had all failed to give her relief. I here point out that esophoria at reading distance gives rise to much more trouble than exophoria, which appears to be exceedingly common and usually gives rise to no discomfort. Indeed, the position of rest as indicated by the Maddox wing test for reading distance does not occupy the middle of R_c but is, I think, always on the divergent side in normal cases.

Exophoria.—In this condition the point 0 in the dia-

gram (i.e., the position of parallelism of the fixation lines for distance) lies to the right of the *region of comfort*, and all that is necessary is to shift the region of comfort to the right so that its right edge just touches the point 0.

Suppose that R_c for distance when the correction is worn is 6 ∇ ranging from $-6\ \nabla$ to 0, his region of comfort must range from $-4\ \nabla$ to $-2\ \nabla$, hence he must have a $-2\ \nabla$ prism (base in) before each eye, to ensure comfort for distance.

$-6\ \nabla$ $\qquad$ $-4\ \nabla$ $\qquad$ $-2\ \nabla$ $\qquad$ 0

The formula for all cases of exophoria would be

$$\text{Max.} - \tfrac{1}{3} R_c = 0 - 2 = -2\ \nabla$$

Exophoria for distance will probably call for correction, but exophoria for reading, as said above, will rarely give rise to discomfort. A slight undercorrection of hypermetropia is often given for this error so as to engage more accommodation which will facilitate the exercise of convergence, but I do not recommend this procedure. Unless the exophoria be very great, squinting exercises are almost sure to cure the condition. If the range of accommodation be also defective for the patient's age, I always urge the procedure mentioned on *p*. 21. If there be no subnormal accommodation, let him try first with his finger brought closer and closer to his nose until it doubles. If he can do this easily when brought to his *punctum proximum* without doubling, let him wear strong convex glasses and exercise his convergence when viewing a vertical needle held closer still. These exercises should be undertaken three times a day when not fatigued, say after meals; they need not last longer than two or three minutes. Relieving prisms should not be given until it is found that this procedure is useless.

I should not hesitate to operate in any case that required a high prismatic correction for symptoms of exophoria or esophoria. It is necessary in all cases for operation to

distinguish between an overaction of the externi or interni and a weakness of their opponents; the distinction is easy, as will be seen from the following table :—

	At a Distance	Near
Exophoria—		
Diverging power in excess ..	Great	Slight
Converging power weak ..	Slight	Great
Esophoria—		
Converging power in excess ..	Slight	Great
Diverging power weak ..	Great	Slight

If in exophoria the diverging power is in excess, the exophoria will be greater at a distance than near, and so on. For excessive power a tenotomy would be indicated, but if the muscles are weak, an advancement operation should of course be performed. A warning, however, should be given not to operate unless the extent and situation of the range of convergence has been made out both for distance and for near work. If this is neglected, it may be found that after correcting a pronounced esophoria for close work one has produced a serious exophoria for distance.

Absolute Maximum of Convergence.—As the near point of convergence is usually about 4 cm. from the bridge of the nose, and as an error of only 2·5 mm. in this situation entails an error of 2 ∇ in the amount of convergence for each eye, it will be seen that some special precautions must be taken to obtain anything like an accurate result. However, as I have never found its determination of the slightest use, and as a different result is obtained each time the patient is examined, I now never waste time over this test.

Points to remember about Convergence Defects :—

1. Always correct hyperphoria and all refractive errors before testing for this variety of heterophoria.

2. Never correct unless symptoms call for it.
3. For *esophoria* give Min. + $\frac{1}{3}$ R_c.
For *exophoria* give Max. — $\frac{1}{3}$ R_c.

The symbols Min. and Max. are to be understood as meaning the *relative* minimum and the *relative* maximum convergence found at the distance for which the glasses are to be used.

4. If the relative convergence range is defective, squinting exercises should be enjoined in exophoria; exercising prisms (abducting) in esophoria.

5. In all cases of noncomitant heterophoria it is well to give iodine (*p.* 97) a trial, as prisms can only give relief when a small area of the glass is used. Of course relief from all symptoms will be obtained at once by wearing a shade over one eye, but this condemnation to monocular vision can hardly be regarded as treatment for the condition.

These rules are such that the prisms ordered are the weakest that are likely to relieve the symptoms, so, if found necessary, one need not hesitate to order rather stronger ones. If none but prisms of 7 ▽ or higher, afford any assistance, an advancement operation must be considered.

The method that I have just given is not that usually recommended; it is customary to measure the absolute range of convergence, and then consider that not more than $\frac{1}{3}$ of this total range can be exercised continuously without fatigue. This sounds a simple rule, but as I have pointed out in describing the test for the maximum convergence, there are very great difficulties in finding the total range. Further, I consider the rule illogical, as the amount of convergence that can be exercised depends entirely upon the amount of accommodation that is exerted at that time. In fact, it must be the relative, not the absolute range that should be considered.

I admit that my rule is only empirical, and that some patients may require rather more than $\frac{1}{3}$ of their relative range to enter into the calculation, but I have found the method exceedingly useful in practice, and I think that the following charts will justify my contention.

It is beyond the province of this book to deal with the subject of squints, but I may remind the reader that if esophoria at a distance is present, it is well fully to correct any hypermetropia and rather to undercorrect myopia.

RELATION OF CONVERGENCE TO ACCOMMODATION.

Donders was the first to study this relation, and to give charts for cases of hypermetropia, emmetropia, and myopia, showing how much they differed from each other. Nagel published several charts of great interest, and by introducing the metre angle, simplified their construction and made them much more readily intelligible. Few people have paid much attention to them since, so I here reproduce some of the charts that I have made out, as I feel that more light is thrown on the proper correction of esophoria and exophoria by the study of this relationship than in any other way.

We will begin by considering the first chart (*Fig.* 10), which may be taken as a normal chart of a person with ·5 D of hypermetropia.

The figures along the horizontal line to the right indicate the metre angles of convergence exercised; thus 0 denotes no convergence, or parallelism of the fixation lines, 3 denotes 3 m.a., or convergence of the fixation lines to a point $\frac{1}{3}$ metre distant. The space to the left of the point 0 denotes negative convergence or divergence. We see at once from the figure, that Dr. Campbell could diverge 1 m.a. and could converge 15 m.a.

The vertical figures under the heading "accommodation" may require some explanation; the figures represent

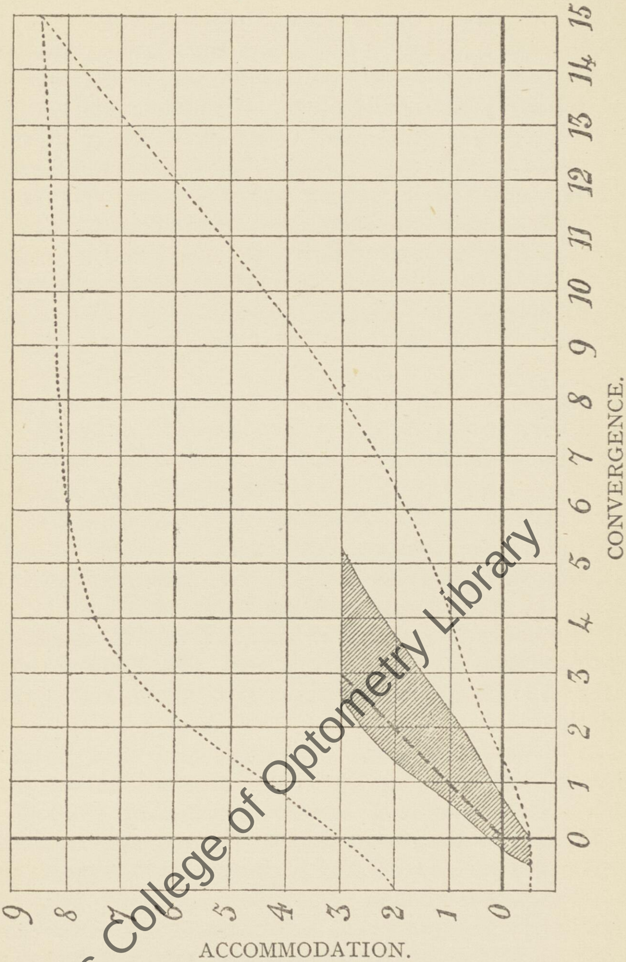


Fig. 10.—Chart of Dr. Mabel Campbell (late House Surgeon at the Newcastle-on-Tyne Eye Infirmary) showing an excellent range of convergence. It is a little peculiar in showing no exophoria at $\frac{1}{3}$ m., the range at A 3 C 3 being from -3 to $+5$ (0 to $+8$ on the chart). It is more usual to find the negative part of the range at this distance greater than the positive part.

the reciprocals of the distances for which the eyes are accommodated; thus 0 represents the state of the eyes when accommodated for infinite distance, 3 when accommodated for $\frac{1}{3}$ metre, and so on. While her fixation lines were diverging 1 m.a., Dr. Campbell could see clearly with either + ·5 D or — 2 D, i.e., she could exert accommodation sufficient to see an object $\frac{1}{2}$ metre distant, so the upper dotted line begins at the point marked 2. When her fixation lines were parallel, she could see clearly with either a + ·5 D or a — 3 D lens, hence the upper dotted line rises to the point 3, while the lower dotted line continues at the level of ·5 below the horizontal line.

When the test object was held at $\frac{1}{3}$ metre, the diverging prisms that could be overcome showed that parallelism could be momentarily attained, and also converging prisms of such strength that they corresponded to a total convergence of 8 m.a., while distinct vision of the test object at $\frac{1}{3}$ metre was still maintained.

In this way the relation of the two functions has been traced, and is represented by the chart, showing that the maximum of convergence is 15 m.a., and that the maximum of accommodation is denoted by 8·5, which, when added to the ·5 D of hypermetropia, represents 9 D of accommodation.

The horizontal lines evidently give the relative convergence for each dioptre of accommodation exercised. Thus at the point 3 (representing + 3·5 D of her accommodation) her range of convergence was from 0 to 8 m.a.; at 7·5 (+ 8 Ac.) her range of convergence was from 4 m.a. to 13·5 m.a.

Similarly, the range of accommodation to a given convergence is given by the vertical lines. Thus, when exerting 4 m.a. of convergence, her range of accommodation is denoted by the line from 7·5 to 1.

It is obvious, then, that the amount of convergence that

can be exercised depends upon the amount of accommodation that is exerted at the same time. Consequently, I contend that it is only the relative range of convergence that can be considered in treating defects of convergence.

Some, laying stress on the fact that in many cases faulty tendencies, as revealed by the rod test, do not give rise to any symptoms, are inclined to disregard them in all cases, and confine their attention to the exact correction of refractive errors. Others urge the correction of every anomaly as soon as it is discovered, attributing to it almost any ache and pain, or indeed anything else that the patient may suffer from. Clearly, if the relation between the two functions is unfitted for present requirements, and if there is no sufficient faculty of adaptation that can be brought into play by training, we should make the glasses suit the patient, instead of vainly attempting to make the patient suit the glasses.

To illustrate what I mean by the faculty of adaptation, I append the chart of an active man of 24 who came to me for glasses (*Fig.* 11). I found simple hypermetropia of + 6 D; but his manifest binocular hypermetropia was only + 3 D. When lenses stronger than + 3 D were given him his eyes diverged. In fact, this hypermetrope had learnt to accommodate without converging, just as most myopes learn to converge without accommodating. The chart shows what discomfort the full correction would cause him until he could adapt himself to the new conditions. Fortunately, most patients are able to adapt themselves to the glasses ordered them, even though these embody a full correction or something very near it. Thus, although I ordered for my patient + 5 D, he only complained of discomfort for the first few days, and then professed himself to be much pleased with the glasses. On his second visit, a fortnight afterwards, I found that instead of diverging when wearing the + 5 D correction, he could not only maintain parallelism of his

fixation lines, but he could overcome adducting prisms of 4·5 ▽ before each eye (i.e., convergence of 1·5 m.a.).

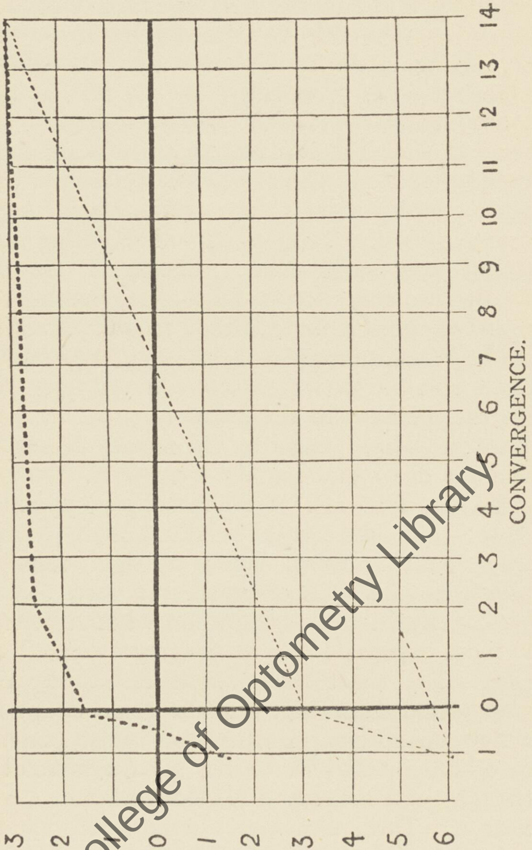


Fig. 11.

It would have been worse than foolish to have ordered prismospheres for this patient when he had such good

adaptive power, but some patients will be found who have no such capacity for adapting themselves to new conditions. This is the reason why new spectacles are so often troublesome at first.

When hypermetropes cannot acquire the art of accommodating without converging, the familiar concomitant squint occurs, and in some cases it will be found that the correction of the hypermetropia at once cures the squint. As this faculty of adaptation varies so much in different people, it appears that the peculiarities of each case must be considered before ordering prismospheres.

Now the requirements of most people will be satisfied if they can see clearly and without discomfort both distant objects and those at $\frac{1}{3}$ metre. Clear binocular vision will be possible if the diagonal from 0 to 3 lie within the figure bounded by the dotted lines, and possible only on this condition; but this does not necessarily imply comfort. What then defines this *area of comfort?*

The Area of Comfort.—From a careful examination of the notes and charts that I have taken of different patients, I have come to the conclusion that this area occupies about the middle third of the relative ranges ✓ between the limits of 0 and 3 m.a. of convergence. Above 3 m.a. this definition does not hold good, but we need not trouble ourselves about the discomfort of patients who persist in holding books too close to them when provided with proper glasses. In the next charts, this area of comfort is shaded; whenever I have found the diagonal (0 to 3) to lie outside this area of comfort, I have heard complaints of asthenopia.

Numerous other suggestions have been made as to the principle on which corrections should be given.

I have mentioned Landolt's suggestion that $\frac{1}{3}$ of the absolute range of convergence can be exercised continuously without fatigue, and I have given my reasons against

its adoption. I have tried this method, and have found that it has frequently failed to give relief.

Some have aimed at putting the absolute far point of convergence in its normal position. Others have tried altering the position of the absolute near point. Neither of these methods commends itself, as the total range of convergence may be anything from 9 m.a. to 16 m.a. without giving rise to any symptoms.

My definition of the area of comfort was published in the *Ophthalmic Review* in 1891, and since that time I have seen nothing to alter in my original communication. Some of my original diagrams are given below.

How to place prisms & lenses To shift The Zone of Comfort

It is clear that, by prisms or by operative interference, we can alter the position of the zero line of convergence and the diagonal line as we will. By ordering abducting prisms of 1 m.a. we can displace these lines towards the left, or with adducting prisms we can displace them towards the right.

Similarly, with the help of spherical lenses we have complete control over the position of the horizontal zero line; convex lenses lower, concave lenses raise it.

The principle that I adopt is sufficiently clear from a study of these diagrams. Whenever the patient has not sufficient adaptive power to suit his new correcting glasses, after a sufficient trial, I so alter them that the diagonal from 0 to 3 falls within the area of comfort.

Examples.—1. Mrs. F., a neurotic emmetrope, aged 25, suffered from asthenopia that was found to be due to an abnormal relative range of convergence. Her chart (*Fig.* 12) shows that with her accommodation relaxed she could diverge 2 m.a. and could also just maintain parallelism. When 3 D of accommodation was exerted her range of convergence was from − ·5 m.a. to 4 m.a. The diagonal was about ·5 m.a. on the positive side of her area of comfort. She had been given + 1 D glasses for reading, which only aggravated her difficulty: this can be easily seen from the chart, as the + 1 D lenses would practically lower the zero horizontal line and the diagonal line one step, so that when exercising 2 D of accommodation she would have to exert her utmost limit

of convergence to see at $\frac{1}{3}$ metre. Clearly -1 D would have been much more serviceable. I ordered her simple abducting prisms ($-1°$ d.) for constant use. These have been quite satisfactory in relieving her of pain, and, what is perhaps better evidence of their

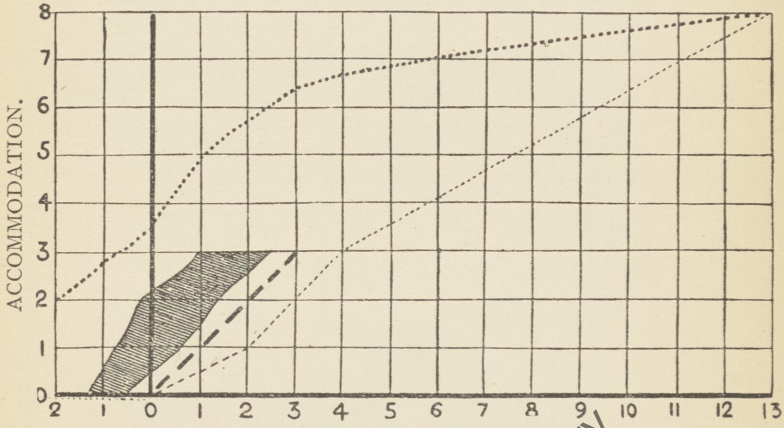


Fig. 12.

benefit, the puffiness of the lids and the conjunctivitis have disappeared and shown no tendency to recur. The abducting prisms ordered, just brought the diagonal within the shaded area of comfort.

2. The next chart (*Fig.* 13) is that of a man, aged 29, with about ·5 D of hypermetropia, who had a very contracted range of convergence. At first, finding that he had 7 D of accommodation, I thought that reading glasses ought to be quite unnecessary, although he had obtained some relief from wearing + 1 D reading glasses. The chart explains this observation, for on lowering the zero horizontal line one step, it is seen that the summit of the diagonal would be in the centre of the area of comfort, being at the point of intersection of 2 of accommodation and 3 of convergence. This case undoubtedly ought to have been given squinting exercises or exercising prisms to increase his convergence range, but as he absolutely refused to undertake orthoptic exercises I gave him adducting prisms of 1° d. associated with the correction of his refractive errors. The result has been completely satisfactory; although I feel sure that, with exercise, it would have been unnecessary for him to wear glasses at all. It is clear from the chart that

if immediate relief were required, adducting prisms of 1 m.a. (+ 1° 43′) should have been ordered, but part of the esophoria was intentionally left to be corrected by the patient's own efforts.

When tested with a Maddox rod, the line of repose almost exactly bisected the area of comfort with the test light held at different distances, from 6 metres to $\frac{1}{3}$ metre; when, however, the test light was held at 6 metres and his accommodation excited by concave glasses, the line of repose occupied the position shown by the nearly vertical dotted line from 1 m.a. to 1·5 m.a., showing

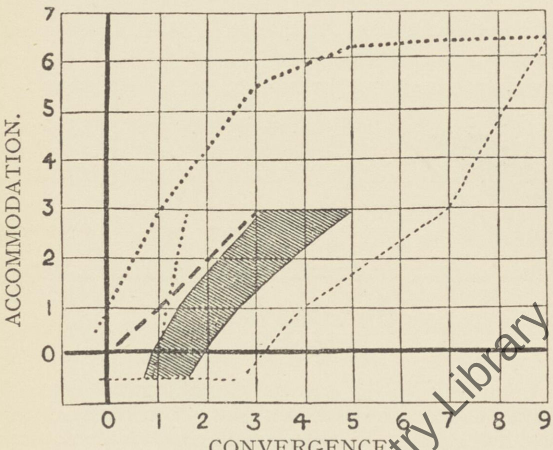


Fig. 13.

the very great influence that the estimate of distance had upon this subject, though in my own case its effect is hardly noticeable.

3. The next chart (*Fig.* 14) is very peculiar; it is that of a lady of pronounced neurotic type, who had been subject to headaches all her life, true migraine and occasional attacks of giddiness. These attacks came on when she looked at distant objects quite as much as when she attempted to do needlework. Her error of refraction was R + ·25 D sph. + ·5 D cyl. ax. 130°. L + ·5 D sph. + ·5 D cyl. ax. 10°.

She had visited many distinguished specialists, who had all found practically the same error of refraction. As will be seen from her chart, she had 5 D of accommodation, which corresponded with her age, 39. Her range of convergence was from − 4 m.a. to 7 m.a., and it is seen that, with glasses, the diagonal is outside the

area of comfort for distance; while it is only just within this area at reading distance. Clearly an advancement of her internal recti, while correcting her exophoria for distance, would give her most troublesome esophoria for reading. The line of repose almost bisected the area of comfort, showing exophoria for distance and esophoria for reading.

I ordered abducting prisms of 1° 30′ to be added to her distance glasses, and + 1 D added to the distance correction (but without prisms) for near work. This was wrong, but at that time I knew very little about the subject; from the chart I ought to have given her abducting prisms of 3 ∇ to both distance and reading glasses, and the addition should have been + ·5 D. I now would

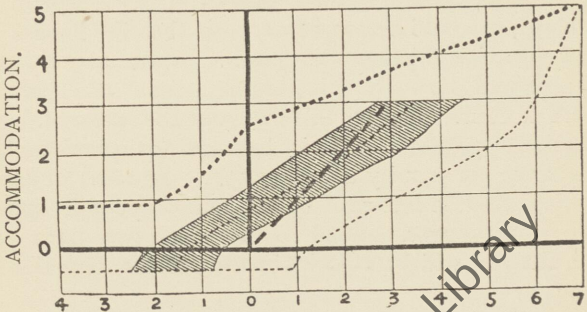


Fig. 14.

have ordered her bifocals, the field for distance lens being a toric prismosphere on a − 6 D base would have served, as the lens required was of such low power. It will be seen that with such a lens for distance, the diagonal line would be displaced one step to the left and half a step downwards, whereas when reading the upper extremity of the diagonal would be displaced one step downwards and one step to the left so that it would lie almost in the centre of the area of comfort for that region. The result, however, of the faulty prescription that I gave was very fairly satisfactory, as she was able to continue reading and working without fatigue, and was almost entirely free from her old headaches.

All the above cases were entirely free from any trace of hyperphoria, and I submit that they afford good evidence in support of my contention as to the definition

of the *area of comfort.* It will be seen that the directions given on *pp.* 121–3 are really only another way of expressing what is shown more fully in these charts.

In conclusion, I would say that few patients will require such a complete examination as is here suggested.

CYCLOPHORIA.

Very occasionally it will be found that symptoms arise from the torsion of one or both eyes about an antero-posterior axis. The condition is very readily recognized with a double prism. Two prisms of 2 ▽ united at their bases, are held before one eye, so that the horizontal line of junction bisects the pupil. When a horizontal line is viewed through this double prism, two parallel lines are of course seen by this eye, and the horizontal line in its normal position—that is, between the previous two parallel lines—is seen by the other eye. In cases of cyclophoria, the middle line seen by the naked eye will appear tilted and no longer parallel to the other two lines. Of course, the eye will be twisted in the reverse direction to the line.

A slight amount of cyclophoria is often found in near vision, but it rarely occasions any symptoms. Savage, of Nashville, is perhaps the greatest authority on this difficult subject; I would refer the reader to his papers and to Maddox's *Ocular Muscles.*

There are two forms: comitant, and noncomitant or paretic. That due to a paresis of an isolated muscle is easily recognized; e.g., a paresis of the right superior oblique would show the greatest intorsion of the false image in the right inferior area of the patient's field and the greatest depression of the false image in the left inferior area of his field, while in the upper part of his field there would be no diplopia. For this form, if seen early, the iodine treatment (*p.* 97) would almost certainly be successful.

If the cyclophoria be comitant, the condition of the

patient is most pitiable; persistent headache and vertigo are the prominent symptoms. If there be intorsion of vertical lines, the houses on either side of the street appear to be falling upon him, and little can be done for his relief except covering one eye. Two concave cylinders, the right set at axis 45°, and the left at axis 135°, may be found to enable him to see vertical lines normally, but horizontal lines will then appear worse than before, and of course a considerable refractive error is entailed. The only other optical suggestion that I can make for his relief is to try two prisms, both the right and left set base out, and for extorsion tilt the upper borders of the prisms away from the eye (vice versa for intorsion), i.e., rotate the prisms round the base-apex line as axis. This clumsy suggestion might be of use in some cases.

A surgical enthusiast might obtain a brilliant success by raising the insertion of the external rectus and lowering the insertion of the internal rectus of the intorted eye.

Cyclo phoria treat,

Less heroic measures seem to me to be preferable and Duane's suggestion sounds eminently reasonable, viz.: To use two of Maddox's rods, one before each eye, and gradually to rotate them in opposite directions while endeavouring to keep the line of light from doubling. This will exercise the muscles that tend to twist the eyes about their antero-posterior axis.

Cyclophoria only for near work may very frequently be discovered; indeed, if the object be very close to the eyes Meissner has shown that an actual extorsion of the eyes normally occurs. I do not think that such cyclophoria ever causes any symptoms or requires any treatment.

In some rare cases of oblique astigmatism, it will be found that the axis of the cylinder must be set at one angle for near vision and at a different angle for distant vision, or troublesome symptoms of cyclophoria will occur. In such cases two pairs of spectacles must be ordered, one for reading and the other for distance.

NOTANDA.

These are not to be regarded as reliable aphorisms, but rather as suggestions to be borne in mind for confirmation or refutation by further experience. I personally have found them useful.

1. Conjunctivitis or blepharitis may be due to uncorrected hypermetropia or astigmatism.

2. If there be no redness of the eyelids, a hypermetrope will not usually complain of his eyes but of headache, which will be aggravated by close work.

3. Evening (or morning) frontal headache suggests uncorrected hypermetropia or astigmatism of low degree; morning ptosis will often be observed when the reading correction is insufficient. Morning headaches suggest eyestrain but they require investigation; it would be very interesting if they were found to be associated with some special refractive or heterophoric error.

4. Giddiness, especially when it ceases on closing one eye, suggests heterophoria or paralysis of one or more ocular muscles.

5. Aching eyes suggest myopia or insufficient convergence.

6. Pain at the back of the eye coming on *immediately* on attempting close work is neurotic. A blister, painting the brows with iodine, or anything that distracts the attention will be effective.

7. Pain after reading for ten minutes or so suggests a change of glasses.

8. Agonizing pain in one eye occurring on first waking, when nothing abnormal can be discovered on examination two or three hours later, suggests a corneal bulla. At

night a bulla may be seen causing a little discomfort; on first opening the eyes in the morning the subepithelial bulla will be torn, exposing the superficial corneal nerves. In two hours or so the raw surface will be re-covered, and no abnormality will be seen. In herpes, on the other hand, puncturing the vesicles will relieve the pain.

9. Pain in a normal eye may be the prodromal symptom of ophthalmic herpes, but it is more probably neurotic.

10. A sensation of grit in the eye of elderly people may suggest either lithiasis or spasmodic entropion, but it is not unusual in other conditions.

11. Torticollis or even lateral curvature of the spine may be of ocular origin; perhaps most frequently it is found in paralysis of the superior oblique muscle.

12. Deterioration of eyesight in a fortnight or so without pain suggests tobacco amblyopia (or retrobulbar neuritis if the loss be confined to, or greater in, one eye). The patient will say that he sees better in a *dim* light, and his appreciation of Light Difference (L.D.) will be far more diminished than his appreciation of Light Minimum (L.M.). In early glaucoma L.M. suffers more than L.D.

13. In cases of anisometropia test the patient when wearing the trial lenses with the Maddox rods for eso- or exophoria; if the red line keeps shifting about, try if adding ·25 D or ·5 D to either eye makes the red line steadier. If + ·25 D added to the right lens steadies the red line, either the right eye requires the addition of + ·25 D or the left eye that of — ·25 D.

FRAME FITTING.

Although this is the business of the optician, it is as well that the ophthalmic surgeon should know something about the subject. A well-fitting frame will hold the lenses in the proper position, and will be comfortable to wear. If the lenses are not ordered to be decentred,

the optical centres should be traversed normally by the visual lines of the eyes behind them.

Asymmetry of the Face.—The two sides of the face are never exactly alike; there are frequently slight differences in the distances of the eyes from the centre of the nose, and even in the heights of the eyes and of the ears. Allowance must be made for all these points with lenses of high power. If the eyes range up or down or to one side behind the lenses, there is added a cylindrical effect owing to the obliquity of the lenses to the visual lines, and a prismatic effect owing to the virtual decentration. When the lenses are of equal strength, the prismatic effect is immaterial, as each eye is similarly affected, but the cylindrical effect may be troublesome and, if high powers are used, necessitate periscopic lenses. If there is a high degree of anisometropia, the prismatic effect may be so annoying as to force the patient to move his head rather than his eyes.

For Reading.—If the eyes are depressed for close work $13\frac{1}{2}°$, the lenses should be tilted downwards $13\frac{1}{2}°$ by bending the legs of the spectacles so that the visual lines shall traverse them normally; in this way the cylindrical effect by oblique refraction will be obviated. If the patient be hypermetropic, the optic centres of the lenses should be lowered 6·5 mm. (i.e. k tan $13\frac{1}{2}$ when $k = 27$ mm.) either by raising the bridge of the spectacles 6·5 mm. or by decentration in order to obviate the increased depression due to the prismatic action (prisms bases up) of the convex lenses. In the case of myopes, as we wish to avoid depression of the eyes it is better not to lower the centres, as in that case if the object be $13\frac{1}{2}°$ below the horizontal plane the eyes will not be depressed to that amount, for now the lenses as they are concave act as prismospheres with their bases down. If there be any difference in height of the two eyes, this will require the needful adjustment of the optic centres of the two lenses.

In addition for close work the eyes will converge, and for really accurate work the distance (m) of each eye from the median line must be measured. In a child m may be 25 mm. or more, in an adult 30 mm. to 32 mm. or even more; then, again, the work may be at a distance of 25 cm., 30 cm., $33\frac{1}{3}$ cm. or more from the plane of the spectacles, so in this table is given the amount (l) the lenses should be decentred or displaced inwards to allow the normal convergence at these distances. On making a diagram it will be seen from the similar triangles that

$$\frac{l}{27} = \frac{m}{p + 27}$$

	p			
m	25 cm.	30 cm.	$33\frac{1}{3}$ cm.	
26 mm.	2·65 mm.	2·15 mm.	1·95 mm.	
28 mm.	2·73 mm.	2·31 mm.	2·10 mm.	l
30 mm.	2·92 mm.	2·48 mm.	2·25 mm.	
32 mm.	3·12 mm.	2·64 mm.	2·40 mm.	

Of course if any troublesome esophoria has been noted convex lenses must be decentred outwards, and concave lenses inwards from this position; it is well to note on the prescription the exact distance of each eye from the median line.

Raphael's transparent 'face rule' is a handy little instrument for roughly making these measurements, though it is based on the assumption that the eyes are symmetrically placed with regard to the median line. On the lower border of the face rule there is a notch which is placed over the nose, and the position of the pupils is given by vertical lines that intersect them numbered 50 to 70 mm. If each pupil (when a distant wall is viewed)

is bisected by the line denoted 62, the pupillary distance (P.D.) is 62 mm. If the left pupil is at the line 62, while the right pupil is at line 66, for the left pupil $m = 31$ mm., and for the ıight pupil $m = 33$ mm. In making this measurement care must be taken that the eyes are directed straight forwards at a distant object.

At the same time that P.D. is determined the distance between the temples may be estimated by reading the number (from 100 to 130) at the right lower border of the rule. One is inclined to over-estimate the temple width, especially if the observing eye be not exactly opposite the marked figures ; it is usually about 45 mm. greater than P.D.

To measure the position of the apex of the bridge use the upper border of the rule, turning it upside down ; place the notch marked 0 just where the bridge of the spectacles will be situated, and read on the lines on either side how many millimetres the mid-pupillary line is above or below the horizontal line touching the apex of the bridge.

In the above account of the use of the face rule, I have given the method for which the instrument was designed ; for accurate work we should find the distance between the points on the corneæ where the corneal images were formed, and not the distance between the two pupils. However, if the distance of each pupillary centre is found by this method, and the distance of the corneal image (*p.* 100) from this point be found, the correction can easily be made. Suppose that in a non-squinting hypermetrope the corneal image were found to be 2 mm. to the inner side of the pupillary centre in each eye and the P.D. were found as above to be 64 mm., the real distance apart of the corneal images would be 60 mm. and the required m would be 30 mm.

The spread of the bridge of the nose is also indicated on the face rule by four different notches numbered 2, 3,

4 and 5 ; it is only necessary to see which of these fits the patient's nose best, and note the number to the optician. This and indeed most of these measurements will be well carried out by a reliable optician, and it is usually only for bedridden patients that this somewhat tedious business will be necessary.

Size of the Spectacle Eye.—There are six recognized oval sizes which are numbered as above, and their dimensions in millimetres are given below.

000	00	0	1	2	3
41×32	40×31	39×29	37×28	36×25	34×25

Circular lenses are unscientific, as a wide lateral range of movement is far more important than a vertical range unless the spectacles are required for shooting. As circular lenses may easily be rotated in their rims, unless some special device is used, they are ill adapted for astigmatic cases.

Testing the Spectacles.—The rims should be in the same plane, so that a flat rule should touch simultaneously both sides of each rim if the lenses were absent ; it is generally easy to see if this would occur even when the rims are holding the lenses.

The hinges for the legs and the connections of the bridge with the rims should be in one straight line. This is very easily tested by holding a straight edge on the facial aspect of the glasses just under the legs, and seeing if the bridge connections also touch the straight edge. The actual lenses should be tested by neutralization, to see if they agree with the prescription.

For finding the position of the optical centres of the lenses and the exact axis of the cylinders Hamblin's lens-centring machine is most useful. There is no difficulty in testing the reading part of bifocal lenses with this instrument, though it is extremely difficult to do without its help.

CHAPTER IV

OPTICAL SECTION

A SOUND and thorough knowledge of elementary optics is absolutely necessary for the explanation and comprehension of the rules and formulæ given in the preceding chapters, and for the solution of the numerous other problems that may present themselves to the practitioner. As the standard works on optics will hardly meet his requirements, I have thought it well to give a brief résumé of the principal problems that arise, and the method of their solution.

I shall assume in this section that the reader is familiar with the elements of the subject.

The Directional Meaning of Signs.—

In all elementary analytical text-books, an origin O is given, through which vertical and horizontal lines are drawn; all lengths to the right of the vertical line and all heights above the horizontal line are considered positive, while lengths to the left of the vertical line and heights below the horizontal line are considered negative. In optics, however, there is no fixed point that is suitable to regard as the origin. In all thick lenses, and in all compound refractive systems, there are two points, H′ and H″, called the First Principal (or Unit) point, and the Second Principal (or Unit) Point, which are used when measuring certain important distances. Thus the first focal length f' is the distance F′ H′ measured from the First Principal Focus (F′) to the First Principal Point (H′), while the second focal length f'' is the distance F″ H″ from the Second Principal Focus (F″) to the Second Principal Point (H″). Similarly the object distance p is PH′

from P the object to H′, but the image distance q is QH″ from Q the image to H″.

The notation for directions varies in almost every optical book to which one refers. The method adopted must be consistent in all problems, or erroneous conclusions will be drawn ; that here advocated (e.g., for p, q, f', and f'') is precisely analogous to the convention universally used in mathematics that the radius (r) of a circle is always measured from the centre towards the circumference. If now we regard the direction of the incident light as always positive, there will be no difficulty in giving the appropriate directional signs to the symbols for lengths, whether the positive direction be from right to left as in *Figs.* 15, 16, 18, etc., or from above downwards.

Dioptres.—The first focal distance, F′ H′ or f', of a convex lens of power + 4 D is positive, + $\frac{1}{4}$ m. in length, whereas the second focal distance F″ H″ or f'' is negative and − $\frac{1}{4}$ m. in length. With a concave lens of − 2 D the first focal distance f' is negative, − $\frac{1}{2}$ m. in length, and the second focal distance f'' is positive, + $\frac{1}{2}$ m.

In every case the power of any refractive system is given in dioptres by the formula $D = \frac{1}{f'}$ if f' is measured in metres; when f' is measured in millimetres the expression becomes $\cdot 001\,D = \frac{1}{f'}$. If the final rays emerge into a medium of refractive index μ', while the source of light is in a medium of index μ_0, $\frac{f'}{f''} = \frac{-\mu_0}{\mu'}$, so if the original and the final medium be the same, as in the case of a lens in air, $f' = -f''$.

In systems in which the two unit points, H′ and H″, are very close to each other, as in thin lenses, they may be regarded as coincident in one point H without appreciable error.

The first principal focus F′ is that point from which

incident rays must originate in order that after refraction they may emerge into the final medium as parallel rays. The point F″ is the second principal focus towards which incident parallel rays converge after their refraction by the medium.

Refraction and Reflection.—The index of refraction $\mu = \frac{V_1}{V_2}$ is the ratio of the velocities of light in the first and in the second medium. From this it follows that, as light after reflection is travelling in the same medium but in the reverse direction, its velocity must be the same in absolute amount, but it must have a negative value. Hence the expression

$$\mu = \frac{V_1}{V_2} \text{ becomes in reflection } \frac{V_1}{-V_1} = -1.$$

Consequently, all the optical formulæ relating to refraction at a single spherical surface can be converted into those of reflection at a mirror by giving to μ the value -1.

For instance, $\mu = \frac{\sin\phi}{\sin\phi'}$ becomes in the case of reflection $-1 = \frac{\sin\phi}{\sin\phi'}$, or $\phi = -\phi'$.

This shows that the angle of reflection ϕ' is measured in the reverse direction from the normal to that of the angle of incidence ϕ.

Again, in refraction at a single spherical surface we have

$$\frac{\mu}{q} - \frac{1}{p} = \frac{\mu - 1}{r} = -\frac{1}{f'} \text{ and } f'' = -\mu f'$$

which becomes in reflection at a spherical surface

$$\frac{-1}{q} - \frac{1}{p} = \frac{-1-1}{r} = -\frac{1}{f'} \text{ or } \frac{1}{q} + \frac{1}{p} = \frac{2}{r} = \frac{1}{f'} \text{ and } f'' = f'.$$

The formulæ for thin *eccentric* pencils incident on a single refracting surface are rarely required, but they may be given,—

$$\frac{\mu\cos^2\phi'}{v_1} - \frac{\cos^2\phi}{u} = \frac{\mu}{v_2} - \frac{1}{u} = \frac{\sin(\phi - \phi')}{r\sin\phi'}$$

where u, v_1 and v_2 represent respectively the distances of

the eccentric portions of the spherical surface from the source of light U, from the first focal line V_1, and from the second focal line V_2. The corresponding formulæ for reflection are obtained from this expression by substituting -1 for μ and ϕ for $-\phi'$;

e.g., $$\frac{-\cos^2\phi}{v_1} - \frac{\cos^2\phi}{u} = \frac{-1}{v_2} - \frac{1}{u} = \frac{\sin 2\phi}{-r\sin\phi} = \frac{2\sin\phi\cos\phi}{-r\sin\phi}$$

or $$\frac{1}{v_1} + \frac{1}{u} = \frac{2}{r\cos\phi}, \quad \text{and} \quad \frac{1}{v_2} + \frac{1}{u} = \frac{2\cos\phi}{r}$$

These illustrations are merely given to show that, if the refraction formulæ are known, the reflection formulæ can be immediately deduced from them.

FORMULÆ.—I also assume that the reader knows that in all cases, whether of refraction or reflection, these two outstanding formulæ always hold good :—

$$(1)\quad \frac{f'}{p} + \frac{f''}{q} = 1. \qquad (2)\quad \frac{i}{o} = \frac{f'}{f'-p} = \frac{f''-q}{f''}.$$

Indeed, little more need be remembered for elementary work.

From (1) we find at once that

$$p = \frac{qf'}{q-f''} \text{ and } q = \frac{pf''}{p-f'}$$

that for a single spherical surface

$$\frac{f'}{p} - \frac{\mu f'}{q} = 1, \text{ or } \frac{1}{p} - \frac{\mu}{q} = \frac{1}{f'}, \text{ or } \frac{\mu}{q} - \frac{1}{p} = -\frac{1}{f'} = \frac{\mu-1}{r}$$

From (2) which gives the ratio $\left(\frac{i}{o}\right)$ of the height of the image to the height of the object, the following is at once obtained : $(f'-p)\,(f''-q) = f'f''$. This form will subsequently be found to be of great service.

Now it is true that these formulæ strictly only hold good for incident axial pencils, so that pencils which are incident obliquely to the optical axis of the refractive system must be treated by more laborious methods (eccentric pencils, *supra*) ; yet the eye is an optical instrument through which one can only see an object clearly at which one is looking directly (*p.* 72), so that we rarely need

consider the *eccentric* refraction of the eye, for it is only thin direct axial pencils that are concerned in macular vision.

Professor Gullstrand in Vol. I of Helmholtz's *Physiological Optics* has devoted pp. 261–382 to an obscure and inadequate discussion of imagery, refraction and aberration of the eye, which practically only gives his results, and not the method by which he attained them. On *p.* 286 he states that " the positive direction is always that which corresponds to the way the light goes in the object space," yet on *p.* 359 he contradicts this statement by strongly advocating the extraordinary method devised by Hess : that " the far point distance and the near point distance are both negative when these points are real." He is discourteous to his readers in assuming that they are not familiar with differential methods and he says (*p.* 264) : " in order not to obscure the results by mathematical intricacies the analytical processes will be entirely omitted." The analytical processes are just what the mathematical ophthalmologists wish to see in order to judge for themselves the validity of his reasoning. The data are so indefinite and the observations so conflicting on many points, as, for instance, the different values of the refractive index of each successive layer of the lens, that it appears to me pure waste of time to work out results on such unreliable data.

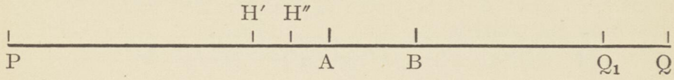
Cardinal Points.—It is most important to understand clearly how to find the position of the Unit Points in any thick lens or in a more complex optical system, such as the eye. Gauss has shown us how to extend the method to any optical system however complex, if the media are bounded by centred spherical surfaces.

Tscherning has shown us how to extend the method so as to include oblique and decentred surfaces by the use of elementary mathematical reasoning. But for the purposes of this little book, whenever cardinal points are used, consideration will be limited to direct axial pencils incident on a centred optical system.

The method used by Gauss is intricate, long, and difficult to follow, but his results for a thick lens may be proved in a much shorter and easier way.

Take the case of a thick meniscus of index μ_1 with both

surfaces, A and B, convex towards the incident light, we will assume that the anterior surface is facing a source of light in air, while the posterior surface is the boundary of a medium of a different refractive index (μ_2), say water. As will be seen, the reasoning is perfectly general, but the line PQ below is a representation of the above condition in a particular case. The symbols f_1' and f_1'' denote the first and second focal distances of the first surface, while f_2' and f_2'' denote the first and second focal distances of the second surface.



Let P denote a point source of light, on encountering the convex surface at A its rays will converge to a point Q_1 distant q_1 or Q_1 A from A, where $q_1 = \frac{pf_1''}{p - f_1'}$ *

This image at Q_1 serves as the object for the second surface at B, its distance $p' = Q_1B = Q_1A + AB = q_1 + t$ where t is the thickness AB. The final image formed by refraction at the second surface will be formed at Q, beyond Q_1 if $\mu_1 > \mu_2$ and the distance QB will be denoted by q.

In every case $p' = \frac{qf_2'}{q - f_2''} = \frac{pf_1''}{p - f_1'} + t$ ....(1)

Now we wish to find the positions of the Unit Points H′ and H″ in this system, so that $\frac{F'H'}{PH'} + \frac{F''H''}{QH''} = 1$, where PH′ is the distance of the First Unit Point H′ from P the object, and QH″ is the distance of the Second Unit Point H″ from the final image Q formed by the whole system. Let H′A be the distance h' of the anterior surface A from H′, and similarly let H″B be the distance h'' of the posterior surface B from H″.

Then PH′ = PA + AH′ = PA − H′A = $p - h'$

* It will be noted that all the symbols in italics denote lengths.

and $QH'' = QB + BH'' = QB - H''B = q - h''$

so $\dfrac{F'H'}{PH'} + \dfrac{F''H''}{QH''} = 1$ becomes $\dfrac{F'}{p-h'} + \dfrac{F''}{q-h''} = 1 \quad \ldots\ldots (2)$

As the equation (2) uses the same values of p and q that were used in equation (1), it only remains to see if by comparing these two equations we may not obtain the required values of h', h'', F' and F''.

(1) may be written

$$t\,(p-f_1')\,(q-f_2'') + pf_1''\,(q-f_2'') - qf_2'\,(p-f_1') = 0\,..$$

or,

$$pq\,(t+f_1''-f_2') - p\,(f_1''f_2''+tf_2'') - q\,(tf_1'-f_1'f_2') + tf_1'f_2'' = 0\,..(1')$$

(2) may be written

$$(p-h')\,(q-h'') - F'\,(q-h'') - F''\,(p-h') = 0$$

or,

$$pq - p\,(F''+h'') - q\,(h'+F') + h'h'' + F'h'' + F''h' = 0\,..(2').$$

On comparing the co-efficients of (1′) and (2′) we find that they are identical if

$$\frac{1}{t+f_1''-f_2'} = \frac{F''+h''}{f_1''f_2''+tf_2''} = \frac{h'+F'}{tf_1'-f_1'f_2'} = \frac{h'h''+F'h''+F''h'}{tf_1'f_2''}$$

If the expression $t + f_1'' - f_2'$ be denoted by K

we see that $F'' = \dfrac{f_1''f_2''}{K}$, $h'' = \dfrac{tf_2''}{K}$, $h' = \dfrac{tf_1'}{K}$, $F' = \dfrac{-f_1'f_2'}{K}$

if $\dfrac{1}{K} = \dfrac{h'h''+F'h''+F''h'}{tf_1'f_2''}$.

On substituting the values just found for each term of the numerator of this fraction, we obtain

$$\frac{1}{K} = \frac{t^2f_1'f_2'' - f_1'f_2'tf_2'' + f_1''f_2''tf_1'}{tf_1'f_2''K^2} = \frac{t-f_2'+f_1''}{K^2} = \frac{1}{K}$$

As the values given above for F', F'', h' and h'' are consistent, they are clearly the true values required.

Cardinal Points of the Eye.—The optical system of the eye consists of a thin corneal meniscus, the aqueous humour, the lens and the vitreous humour, while the retina forms the screen on which the images are formed. With the help of the adjoining table of the ocular constants, it will be an easy matter to find the cardinal points of the corneal meniscus. We will use Gullstrand's

constants, and give the whole procedure in detail. It is necessary first to find the first and second focal distances for the two surfaces of the cornea.

POSITION OF THE SURFACES.

	G.	Ts.
Cornea ant. surf.	0	0
,, post. surf.	0·5 mm.	1·15 mm.
Lens ant. surf.	3·6 mm.	3·54 mm.
,, post. surf.	7·2 mm.	7·60 mm.

RADII OF CURVATURE AND REFRACTIVE INDICES.

	G.	Ts.
Cornea ant. radius r_1	$-$ 7·7 mm.	$-$ 7·98 mm.
,, post-radius r_2	$-$ 6·8 mm.	$-$ 6·22 mm.
Lens ant. radius r_3	$-$ 10·0 mm.	$-$ 10·20 mm.
,, post-radius r_4	6·0 mm.	6·17 mm.
Corneal index μ_1	1·376	1·377
Aqueous} Vitreous} index μ_2	1·336	1·3365
Lens index μ_3	1·386	1·42 total
,, core	1·406	

The above table gives the values of the ocular constants as estimated by the two great living authorities on the subject, Gullstrand and Tscherning. Gullstrand's values have been taken from Helmholtz's *Physiological Optics*, Vol. I. (published 1924), and Tscherning's values are to be found in his *Physiologic Optics* (published 1920). It will be seen that they differ from each other considerably, although the values of the principal focal distances, which will be given (p. 155) are in very fair agreement.

$$f_1' = \frac{-r_1}{\mu_1 - 1} = \frac{7{\cdot}7}{{\cdot}376} = 20{\cdot}4787 \text{ mm.}$$

Now in all cases of refraction at a single spherical surface $f'' = r - f'$; so here $f_1'' = -7{\cdot}7 - 20{\cdot}4787 = -28{\cdot}1787$ mm. Similarly for the second surface,

$$f_2' = \frac{-\mu_1 r_2}{\mu_2 - \mu_1} = \frac{1{\cdot}376 \times 6{\cdot}8}{1{\cdot}336 - 1{\cdot}376} = -170 \times 1{\cdot}376 = -233{\cdot}92 \text{ mm.}$$

and $f_2'' = r_2 - f_2'^{*} = -6{\cdot}8 + 233{\cdot}92 = 227{\cdot}12$ mm.

From the table it is seen that the thickness (t) of the cornea is ·5 mm.,

so K or $t + f_1'' - f_2' = {\cdot}5 - 28{\cdot}1787 + 233{\cdot}92 = 206{\cdot}2413$ mm.

$$\therefore \frac{f'_1}{K} = \frac{20{\cdot}4787}{206{\cdot}2413} = {\cdot}099295, \text{ and } \frac{f_2''}{K} = \frac{227{\cdot}12}{206{\cdot}2413} = 1{\cdot}10123.$$

But $\frac{f_1'}{K}$ or $\cdot099295 = \frac{h_a'}{t} = \frac{F_a'}{-f_2'}$ and $\frac{f_2''}{K}$ or $1{\cdot}10123 = \frac{h_a''}{t} = \frac{F_a''}{f_1''}$

$$\therefore \quad h_a' = ({\cdot}5)\ ({\cdot}09929) = {\cdot}0496 \text{ mm.,}$$
$$h_a'' = ({\cdot}5)\ (1{\cdot}10125) = {\cdot}5506 \text{ mm.,}$$
$$F_a' = (233{\cdot}92)\ ({\cdot}099295) = 23{\cdot}2271 \text{ mm.,}$$
$$F_a'' = -(28{\cdot}1787)\ (1{\cdot}10123) = -31{\cdot}0312 \text{ mm.}$$

It must be remembered that h_a' is the distance ($H_a'A_o$) of the anterior surface of the cornea from the First Unit Point, while h_a'' is the distance of the posterior surface of the cornea from the Second Unit Point; consequently Gullstrand, who takes the distance of each Unit Point from the anterior surface of the cornea, gives $A_oH_a' = -{\cdot}0496$ mm. and $A_oH_a'' = -{\cdot}0506$ mm. But these are the only two lengths that he gives to four decimal places, all the other measurements are given to three decimal places.

Since I, like Tscherning, wish to give the Nodal Points (K′ and K″), which Gullstrand omits, I give my results in the table on *p.* 155 to three decimal places in brackets. I call attention to this minute point, as with Gullstrand's figures the results are inconsistent for the nodal points in the third decimal place, though if he had taken them all to four decimal places there would have been no discrepancy.

Assuming that the corneal curvature remains unchanged after the operation, the table (*p.* 155) may be regarded as giving the cardinal points for the aphakic eye. Similarly

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* It is easily seen that $f''_2 + f_2' = r_2$,

$$\text{for } f''_2 + f'_2 = \frac{\mu_2 r_2 - \mu_1 r_2}{\mu_2 - \mu_1} = r_2$$

by using Tscherning's constants Tscherning's values are found, and these are also given in the table.

In order to find the cardinal points of a complete eye we must add the refractive value of the lens *in situ* to that of the aphakic eye. When combining two systems A and B we proceed exactly as before,

$$K = T + F_a'' - F_b'$$

where F_a'' refers to the first system A, and F_b' refers to the second system B ; T is now $H_a''H_b'$, i.e., the distance of the First Unit Point of the second system from the Second Unit Point of the first system.

Reliable data of the constants of the lens appear to be impossible to ascertain ; post-mortem changes of the lens when removed will invalidate all the results of estimating the values of the index μ_3 which increase from the periphery to the centre. Differences of the curvature of the surfaces will be found also at different ages, indeed this last point seems to depend very largely also on the stature of the owner. As Javal has said, an elephant and a mouse may be emmetropic, but the curvatures will be greater in the eyes of the mouse. We are not told in Gullstrand's long appendix to Helmholtz's *Physiological Optics* what his observed data were, nor what correction he made to obtain results more in accordance with clinical observations. However, he states that it is quite necessary to take account of this increase of the value of μ_3 towards the centre of the lens in assigning the position of its Unit Points. This is reasonable, but we are anxious to know more details.

Tscherning quite frankly tells us that his estimation of μ_3 is that of an imaginary lens with a uniform index, which is in accordance with the procedure of others.

Gullstrand gives the distance (A_oH_b') of the First Unit Point (H_b') of the lens behind the anterior surface of the cornea as 5·678 mm. and A_oH_b'' as 5·808 mm., and as the lens is bounded on both surfaces by media of the same refractive index, F_b' or $- F_b'' = 69{\cdot}908$ mm.

$T = H_a''A_o + A_oH_b' = {\cdot}0506 + 5{\cdot}678 = 5{\cdot}7286$ mm. and

$K = T + F_a'' - F_b' = 5{\cdot}7286 - 31{\cdot}0312 - 69{\cdot}908 = -95{\cdot}2106$mm.

$$\frac{F_a'}{K} = \frac{23{\cdot}2271}{-95{\cdot}2106} = -{\cdot}243955 = \frac{h'}{T} = \frac{F'}{-F_b'},$$

$$\therefore \quad h' = -\cdot 243955 \times 5\cdot 7286 = -1\cdot 3975 \text{ mm.}$$
$$F' = \cdot 243955 \times 69\cdot 908 = 17\cdot 0543 \text{ mm.}$$
$$\frac{F_b''}{K} = \frac{-69\cdot 908}{-95\cdot 2106} = \cdot 73425 = \frac{h''}{T} = \frac{F''}{F_a''}$$
$$\therefore \quad h'' = \cdot 73425 \times 5\cdot 7286 = 4\cdot 206 \text{ mm.}$$
$$F'' = -31\cdot 0312 \times \cdot 73425 = -22\cdot 7847 \text{ mm.}$$

Now as found by this method h' is the distance $H'H_a'$, i.e., the distance of H_a' from the First Unit Point of the system, so

$A_oH' = A_oH_a' - H'H_a' = -\cdot 0496 + 1\cdot 3975 = 1\cdot 3479$ mm.

Similarly, since $h'' = H''H_b''$,

$A_oH'' = A_oH_b'' - H''H_b'' = 5\cdot 808 - 4\cdot 206 = 1\cdot 602$ mm.

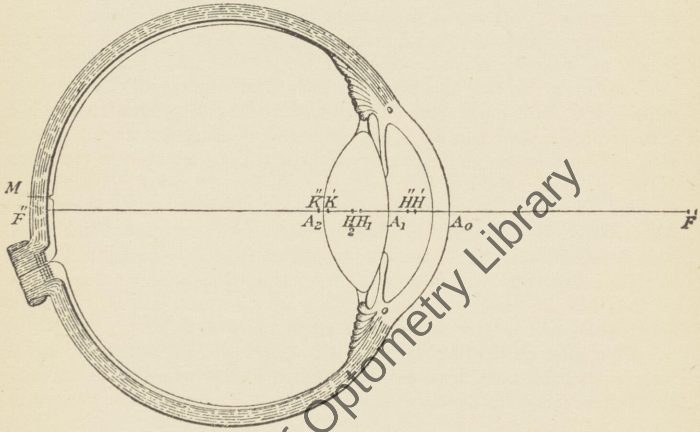

Fig. 15. (After Landolt.)

Reference to the diagram (*Fig.* 15) will make the letters used in the second table for the complete eye quite clear, except that in the diagram the letters H_1 and H_2 are substituted for H_b' and H_b''; the symbol A_o represents the anterior surface of the cornea. I am responsible for all the figures in brackets.

The following table gives in a tabulated form for the

Aphakic and for the Complete eye the results which Gullstrand and Tscherning have published. As will be noticed I make $F' = 17{\cdot}0543$; Gullstrand's figure 17·055 is probably a misprint, for he gives the dioptric power of the eye as 58·64 D which indicates that 17·054 was really the value he found for F', as if F' were 17·055 the value of D would be 58·6338. However, this is an insignificant point, as probably reliance cannot be placed on more than two places of decimals, when the figures are to be regarded as average values. When Tscherning's constants are used, I get his results to two places of decimals, which he wisely considers sufficient.

APHAKIC EYE.

	G.	Ts.		G.	Ts.
A_oH_a'	– ·0496 mm. (– ·050 mm.)	– ·13 mm.	$H_a'H_a''$ $K_a'K_a''$	(– ·001 mm.)	– ·01 mm.
A_oH_a''	– ·0506 mm. (– ·051 mm.)	– ·14 mm.	$H_a'K_a'$ $H_a''K_a''$	(7·804 mm.)	8·21 mm.
A_oK_a' A_oK_a''	(7·754 mm.) (7·753 mm.)	8·08 mm. 8·07 mm.	$F_a'H_a'$ $K_a''F_a''$	23·227 mm.	24·40 mm.
A_oF_a' A_oF_a''	– 23·277 mm. 30·981 mm. (30·980 mm.)	– 24·53 mm. 32·47 mm.	$F_a''H_a''$ $K_a'F_a'$	– 31·031 mm.	– 32·61 mm.
$D_a =$	+ 43·05 D (+ 43·053 D)	+ 40·98 D			

COMPLETE EYE.

	G.	Ts.		G.	Ts.
A_oH' A_oH''	1·348 mm. 1·602 mm.	1·54 mm. 1·86 mm.	$H'H''$ $K'K''$	(·254 mm.)	·32 mm.
A_oK' A_oK''	(7·079 mm.) (7·333 mm.)	7·30 mm. 7·62 mm.	$H'K'$ $H''K''$	(5·731 mm.)	5·76 mm.
A_oF'	– 15·707 mm.? (– 15·706 mm.)	– 15·59 mm.	$F'H'$ $K''F''$	17·055 mm.? (17·054 mm.)	17·13 mm.
A_oF'' $D =$	24·387 mm. + 58·64 D (+ 58·636 D)	24·75 mm. + 58·38 D	$F''H''$ $K'F'$	– 22·785 mm.	– 22·89 mm.

Now which are to be considered the more trustworthy? Tscherning himself almost answers the question. His

constants were taken from one eye which he examined as carefully as possible, but he expressly says, "it is most important to note that it is not to be regarded as an average" (*p.* 40, *Physiol. Optics*). The eye was that of a tall man, and the average value of the radii of the corneal curvatures of tall emmetropes (*cuirassiers*) he found to be 7·8 mm. Under these circumstances I have decided to adopt the values of Gullstrand as the best generally representative values.

The Nodal Points and the Unit Points.—In any complex centred refracting system the method of obtaining the positions of the Principal Foci and the Unit Points has

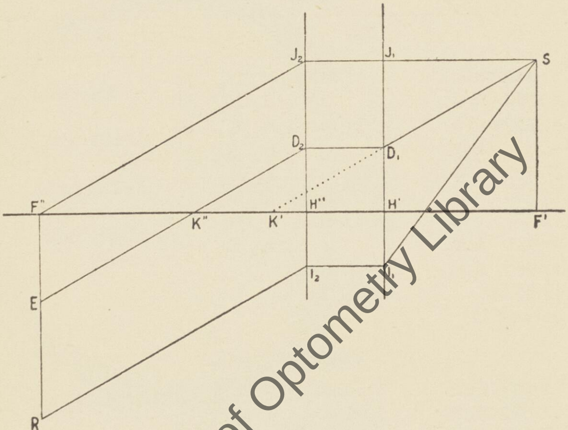

Fig. 16.

just been shown; mark on the axis their positions as in *Fig.* 16, and through these points draw straight lines at right angles to the axis to represent the Focal and Unit Planes. The diagram represents these planes in the eye. Now the directions of an incident ray and its corresponding emergent ray cut the two Unit Planes in two points situated on the same side and at the same distance from the axis.

Therefore if S be a luminous point in the first focal plane, and SJ_1 be one of its rays, viz., that parallel to the axis, it must proceed to J_2 on the second Unit Plane, and then be deflected towards F″. Similarly any other ray such as SI_1 must pass to I_2 and then emerge parallel to J_2F''.

In order to find the Nodal Points we have merely to draw from S a line SD_1K' parallel to J_2F'', and then from D_2 the point corresponding to D_1 draw $D_2K''E$ parallel to J_2F''. Then K′ and K″, the two points on the axis, are the nodal points. For an incident ray SK′ emerges after traversing the system in the direction K″E which is parallel to SK′.

As the sides of the $\triangle$ F′SK′ are parallel to the sides of the $\triangle$ $H''J_2F''$, and as $F'S = H''J_2$, $K'F' = F''H''$ or F''. Also the $\triangle$ $EF''K'' = \triangle D_1J_1S$, as the sides are parallel and as $EF'' = D_2J_2 = D_1J_1$, $K''F'' = SJ_1 = F'H'$ or F'. It is also clear that $K''K' = D_2D_1 = H''H'$. If, then, the positions of the Unit and the Focal Points are known, it is only necessary to mark the points K′ and K″ so that $K'F' = F''H''$, and $K''F'' = F'H'$.

Simplified Schematic Eye.—On looking again at the table of values for the aphakic eye, it will be noticed that the two Nodal Points are so close together (·001 mm.) that no appreciable error will be introduced by considering the two points coincident at one point K which may be then regarded as the centre of a single spherical surface representing the eye. We may then regard

$$r = -7{\cdot}8 \text{ mm.}, \quad F_a' = 23{\cdot}23 \text{ mm., and}$$

$$F_a'' = -31{\cdot}03 \text{ mm.}, \quad \mu = \frac{F_a''}{-F_a'} = 1{\cdot}336.$$

Again, if the difference ·254 mm. between the two nodal points of the complete eye be regarded as negligible, we may consider the complete eye as represented by a single spherical surface of radius

$$K'H' = -5{\cdot}73 \text{ mm.}, \quad F' = 17{\cdot}05 \text{ mm.},$$

$$F'' = -22{\cdot}78 \text{ mm.}, \quad \mu = \frac{F''}{-F'} = 1{\cdot}336.$$

In every system in which the initial and the final media have the same refractive index (i.e., when F″H″ = H′F′), the Nodal Points K′K″ coincide with the Principal Points H′H″.

Fig. 17 shows how useful these points are. To find the image of AB, we join AK′ and draw K″*a* parallel to it, and then either draw AJ″ parallel to the axis and J″F″*a* through F″, or we draw AF′I′ through F′ and draw I′I″*a* parallel to the axis. In either case *a* is the point of intersection of the line with that through the nodal point K″.

Now Gullstrand (Helmholtz's *Physiological Optics*) quite correctly points out that these simple geometrical diagrams do not define precisely the limits of the images of large objects that subtend an angle of some magnitude at K′, the incidence from all but the axial part of the object is oblique, and the nodal and the focal points have changed their position owing to the oblique refraction. But I have already pointed out that the only definite vision of an object is obtained by the very small part of it that falls on the macula; and in viewing a large object one is continually shifting one's gaze from one point to another to form a definite idea of the details of the whole object. It is quite immaterial whether the image that falls on the extra-macular part of the retina is accurately focused or not. Whenever the focal points remain constant, i.e., whenever macular vision is under consideration, the nodal and the unit points remain constant, and reliance can be placed upon them. Gullstrand, however, casts the nodal points and the unit planes (why not the focal points also ?) aside (Vol. I, *p.* 270) "as so much useless ballast." On *p.* 195 *et seq.* in this book, where the problem of Periscopic Lenses will be considered, the vision is eccentric through the lenses, but it is still axial through the eye.

On referring to *Fig.* 17 it can be seen why the points H′ and H″ are called Unit Points, and the planes I′J′ and I″J″ are called Unit Planes, for they are such that any small object placed in the first Unit Plane has its image equal in magnitude and similarly placed in the second Unit Plane. This is their characteristic and distinctive property.

It is clear (*Fig.* 17) that $\frac{i}{o} = \frac{ab}{AB} = \frac{bK''}{BK'}$

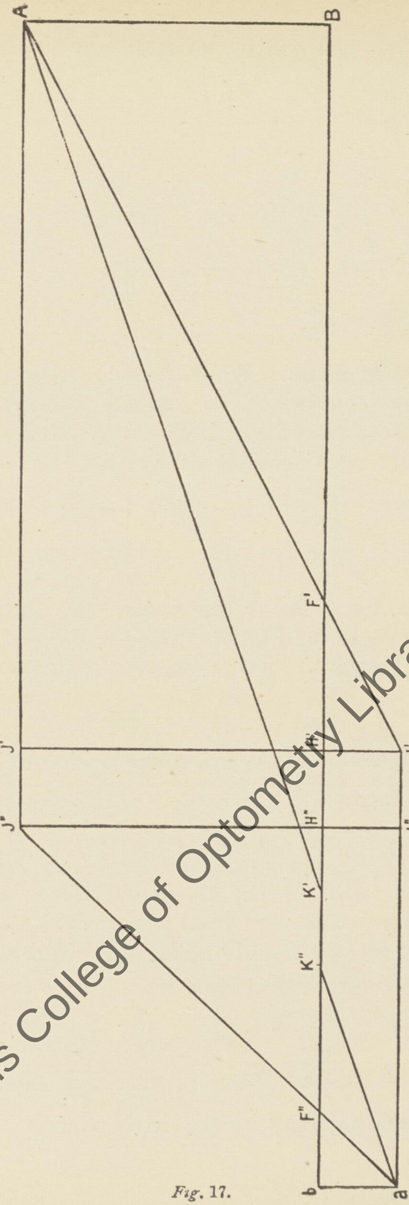

Fig. 17.

Again $\frac{i}{o}$ or $\frac{\mathrm{ab}}{\mathrm{J''H''}} = \frac{\mathrm{F''b}}{\mathrm{F''H''}} = \frac{\mathrm{F''H''} - \mathrm{bH''}}{\mathrm{F''H''}} = \frac{F'' - q}{F''}$;

and also $\frac{i}{o} = \frac{\mathrm{I'H'}}{\mathrm{AB}} = \frac{\mathrm{F'H'}}{\mathrm{F'B}} = \frac{\mathrm{F'H'}}{\mathrm{F'H'} - \mathrm{BH'}} = \frac{F'}{F' - p}$.

Since $\frac{i}{o} = \frac{F'}{F' - p} = \frac{F'' - q}{F''}$, $(F' - p)\ (F'' - q) = F'F''$

i.e., $F''F'' - F'q - F''p + pq = F'F''$.

or $F'q + F''p = pq$ i.e., $\frac{F'}{p} + \frac{F''}{q} = 1$.

Dioptric Formulæ.—When dealing with lenses it is usually more convenient to employ dioptric formulæ instead of those that have been given; but in all such cases *the metre must be taken as the unit.*

We have $\frac{i}{o} = \frac{F'' - q}{F''} = 1 - \frac{q}{F''} = 1 + \frac{q}{F'} = 1 + q\mathrm{D}$

also $\frac{o}{i}$ or $\frac{F' - p}{F'} = 1 - p\mathrm{D}$ $\therefore$ $\frac{i}{o} = \frac{1}{1 - p\mathrm{D}}$

For instance, let a linear dimension (say the height) of an object be 5 cm. and let its distance from a + 5 D lens be 520 cm.; then $o = \cdot 05$ m. and $p = 5{\cdot}2$ m.

$i = \frac{o}{1 - p\mathrm{D}} = \frac{\cdot 05}{1 - (5{\cdot}2)5} = \frac{\cdot 05}{-25} = -\cdot 002$ m. $= -2$ mm. high.

The minus sign shows that the image is *inverted* and real.

From the formula $\frac{1}{p} - \frac{1}{q} = \frac{1}{F} = \mathrm{D}$ it follows that

$\mathrm{D} = \frac{q - p}{qp}$, that $q = \frac{p}{1 - p\mathrm{D}}$ and that $p = \frac{q}{1 + q\mathrm{D}}$.

The dioptric form for the Cardinal Points is very convenient in many cases; the total power (D) of the refracting system must first be found and from D the position of the Unit Points. The total power is

$$\mathrm{D} = \frac{1}{F'} = \frac{t + f_1'' - f_2'}{-f_1' f_2'} = \frac{1}{f_1'} - \frac{f_1''}{f_1' f_2'} - \frac{t}{f_1' f_2'} \quad \ldots \ldots (1)$$

Now as $\frac{1}{f_1'} = \mathrm{d}_1$ the power of the first element in the system always, (1) may be rewritten as

$$D = d_1 - \frac{d_1 f_1''}{f_2'} - \frac{td_1}{f_2'} \quad \ldots\ldots\ldots\ldots(1')$$

and as $\frac{1}{D} = F_1 = \frac{-f_1'f_2'}{K}$, $\quad \frac{1}{K} = \frac{1}{-Df_1'f_2'} = \frac{-d_1}{Df_2'}$

(1) *Lens system in air* is extremely simple, for then d_1 is the power of the first lens, d_2 or $\frac{1}{f_2'}$ is the power of the second lens, and $f_1'' = -f_1'$ and $f_2'' = -f_2'$, so

$$D = d_1 + d_2 - td_1d_2, \quad \text{and } \frac{1}{K} = \frac{-d_1d_2}{D}$$

$$\therefore\ h' \text{ or } \frac{f_1't}{K} = \frac{-td_2}{D}, \quad \text{and } h'' = \frac{f_2''t}{K} = \frac{td_1}{D}$$

(2) *Lens in air before the eye.*—The only alteration from the above (1) is that $f_2'' = -\mu f_2'$, where f_2' is the F' of the eye, so $D = d_1 + d_2 - td_1d_2$, $h' = \frac{-td_2}{D}$, and $h'' = \frac{\mu td_1}{D}$

(3) *Thick Lens.*—Here $f_1'' = -\mu f_1' = \frac{-\mu}{d_1}$, and $f_2'' = \frac{-1}{d_2}$ while $f_2' = \frac{\mu}{d_2}$, and $\frac{1}{K} = \frac{-d_1d_2}{\mu D}$

(This point in thick lenses should be noted; that while f_1'' is numerically greater than f_1', f_2' is always numerically greater than f_2''.)

$$\therefore\ D = d_1 + d_2 - \frac{t}{\mu}d_1d_2, \quad h' = \frac{-td_2}{\mu D}, \text{ and } h'' = \frac{td_1}{\mu D}$$

These three cases cover all the conditions that will be ordinarily encountered. It is known that if two thin lenses, d_1 and d_2, are placed in contact with each other, the resulting power, D, of the combination is equal to their sum $d_1 + d_2$.

From (1) $D = d_1 + d_2 - td_1d_2$, it is clear that if d_1 and d_2 be of the same sign, the term $-td_1d_2$ is equivalent to an addition of a negative (concave) lens to the combination, if d_1 be of opposite sign to d_2 there is an addition of a positive lens to the combination.

Thus if $d_1 = +3\,D$, $d_2 = +5\,D$, and $t = 1$ cm. or $\cdot01$ m.
$D = 3 + 5 - \cdot15 = +7\cdot85\,D$

if $d_1 = -3\,D$, $\quad D = -3 + 5 + \cdot15 = +2\cdot15\,D$

The formula gives us the resulting power of the combination, the reciprocal of $F'H'$ in metres, but it does not tell us where the first or the second focus is; for that we must determine the values of h' and h''.

If now d_2 be positive and the first lens d_1 be placed in the anterior focal plane of the second lens,

$$t = \frac{1}{d_2} \text{ so } - td_1d_2 = - d_1$$

$$D = d_1 + d_2 - td_1d_2 = d_1 + d_2 - d_1 = d_2$$

The addition of d_1 to the original lens d_2 makes no difference whatever to the power of d_2; but it alters the position of the Second Unit Point for $h'' = \frac{td_1}{D}$ and therefore the *position* of the Second Principal Focus; meanwhile the First Unit Point is in its old position for $h_1 = \frac{-td_2}{D} = -t$, i.e., at the second lens at a distance of t from the first lens.

This fact should be verified by experiment; it is of fundamental importance in nearly all the optical problems that will arise, though no mention of it is made in the optical text-books.

Experiment.—A rough experiment can be made with a + 16 D and a + 10 D lens from the trial case. In the absence of an optical bench, a piece of corrugated paper wrapped round the lenses will hold them parallel and approximately 6·25 cm. apart. (The focal length of a + 16 D is 6·25 cm.) A piece of corrugated paper about 14 × 8 cm. is taken with the corrugations lying lengthwise; a small slit is made in the middle of the furrow adjoining one edge through which the tiny handle of the + 16 D lens can be passed, another slit is made in the furrow 6·25 cm. from it for the + 10 D lens. Insert the lenses and wrap the corrugated paper round them so as to keep them parallel and fix it temporarily with two elastic bands or strings, mark the point where the handle of the 16 D projects with the letter B, and the position of the other lens with the letter A. Now with a + 16 D lens form an inverted image of a distant object, say a window, on a screen; note the distance of the screen from the lens, it will be about 6·25 cm., a trifle more if the window be less than 5 m. distant, and note the size of the image. Then hold the combination with the point B 6·25 cm. from the screen,

no image will be formed, yet the lens at A is in the anterior focal plane of D_2 and from our formula we know that its power is + 16 D, and therefore its focal length must be 6·25 cm. On holding a card between B and the screen it will be found that when it is advanced about 3·9 cm. from the screen a sharp inverted image of the window will be formed on it of exactly the same size as that formed by the 16 D lens. Fix the card in this position, and try with a nearer object, say one at 30 cm. distance. With the + 16 D lens an image will be formed when it is 7·9 cm. from the screen; similarly the image of this object will be formed on the card when B and the 16 D lens are in the same plane, i.e., when B is about 4 cm. from the card. The images again will be the same size, so the power of the combination is + 16 D, the Second Unit Point must be about 3·9 cm. on the A side of B, while the First Unit Point H′ must be at B, as will be seen from the formulæ, provided that care is taken to use the metre as the unit (t for instance is ·0625 m., etc.). If now the apparatus is reversed so that the surface A faces the screen no image will be formed at whatever distance it be held. The focal length is still 6·25 cm., but a moment's thought will make one realize that the image is then formed in the plane of A or the d_1 lens, and on looking directly at d_1 one will see it there. This prepares one for the statement that a lens bounded by surfaces of unequal curvature will have a different effect when a different surface is exposed to the incident light. In fact, reversing any optical system changes the second cardinal points into the first cardinal points and the first into the second; the points that were H′ and F′ become H″ and F″.

On substituting a − 10 D lens for the + 10 D lens at A, and repeating the first experiment, it will be found that the card must now be placed 3·9 cm. behind the plane of the screen to receive the image. I give below the cardinal points of the apparatus with these different lenses at A as determined by the formulæ.

A + 10	A − 10
B + 16	B + 16
D + 26 − 10 = 16D	D + 6 + 10 = 16 D
$h' = \frac{-td_2}{D} = \frac{-1}{16} = -\cdot0625$ m.	H′A = − ·0625 m. at B
$h'' = \frac{td_1}{D} = \frac{\cdot625}{16} \approx \cdot039$ m.	H″B $\approx$ − ·039 m.
BF″ $\approx$ ·0235 m	BF″ $\approx$ ·1015 m.

As with this simple apparatus the site of d_1 will not be exactly in the anterior focal plane of d_2 and for other reasons (*p.* 175), the results will be only approximately correct, but they will be quite sufficient to convince one of the principle involved.

Here we have by the addition of a +10 D lens in the anterior focal plane of a + 16 D lens produced exactly the same results on a screen advanced 3·9 cm. as without that glass was obtained on a screen 3·9 cm. further back. This, then, is the ideal method of correcting axial ametropia due to abnormal length of the eye. In hypermetropia (*Fig.* 19) when the eye is too short, we should provide a convex lens placed in the anterior focal plane of the eye of such a power that the image shall be formed on the retina in its abnormal position. When that is done the retinal images will be of the same size as those of the standard eye, and, consequently, all visual tests will be strictly comparable. Similarly in *Fig.* 18 a concave lens placed in the anterior focal plane of the eye leaves the First Unit Point h' in its old position H', but the Second Unit Point has been moved towards the retina so that the Second Principal Focus is at Q instead of F''.

Ametropia and Axial Length.—As ametropia almost always depends upon variations in the axial length of the eye, we must consider what must be the power of the lens which, placed in the anterior focal plane of the eye at F', will correct a defect or an excess (l) of length of the eye.

Fig. 18 is a diagrammatic sketch of a myopic eye that is too long by $F''Q$ or l in millimetres, while the retina of an emmetropic eye would be at F''. Let us assume that for this eye an object must be 100 mm. in front of the cornea in order that a definite image of it may be formed at Q. For viewing distant objects it will be necessary to interpose a concave lens such that incident parallel rays will come to a focus at the *punctum remotum* (*p.r.*). In this case as *p.r.* is 100 mm. in front of the cornea, a lens of — 10 D would serve if it could be placed in contact with the cornea; if the lens were placed at F', which will be about 85 mm. from the *p.r.*, a concave lens of focal length 85 mm. (say about −11·75 D) will be required. In ametropia of any kind the correction for distance is always

that lens which when placed in a suitable position has its second principal focus at *p.r.* If P be the *p.r.* and the lens be at F′, PF′ must be its second focal distance, and F′P must be its first focal distance (f') :
but $F'P = F'H' - PH' = F' - p = f'$,
and $F''Q$ or $l = F''H'' - QH'' = F'' - q$,
so $(F' - p)\ (F'' - q)$ or $f'l = F'F''$,
and if these lengths are measured in millimetres

$$\cdot 001\ D' = \frac{1}{f'} = \frac{l}{F'F''} = \frac{l}{(17\cdot054)\ (-22\cdot785)} = \frac{l}{-388\cdot575}$$

$\therefore\ l = -\ \cdot388575\ D'$ and $D' = -\ 2\cdot5735\ l$.

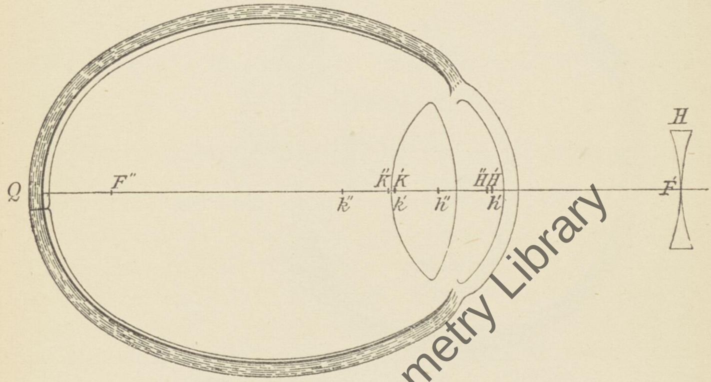

Fig. 18.

As a rough approximation we may say that $l = -\ \cdot39\ D'$, and $D' = -\ 2\cdot57\ l$ where D′ is the power of the correcting lens when placed in the anterior focal plane of the eye, i.e., 15·706 mm. from the cornea.

Examples.—What lens in the first focal plane will be required to correct an eye (1) that is 1·56 mm. too long (AF″ ≏ 25·95 mm.), and (2) one that is 1·17 mm. too short (AF″ ≏ 23·22 mm.) ?

(1) $D' = -\ 2\cdot57 \times 1\cdot56 = -\ 4\ D.$

(2) $D' = -\ 2\cdot57\ (-\ 1\cdot17) = +\ 3\ D.$

The peripheral edge of an optic disc is sharply defined with + 4 D, but except for this rim the rest is ill-defined until a + 1 D is used behind the ophthalmoscope, what is the depth of the cup?

There is − 3 D difference $\therefore l = - \cdot 39 (-3) = 1 \cdot 17$ mm. The base of the disc is depressed 1·17 mm.

This is the method by which all differences of level are measured with the ophthalmoscope; it is, indeed, the absolutely reliable test of a detached retina. For accurate

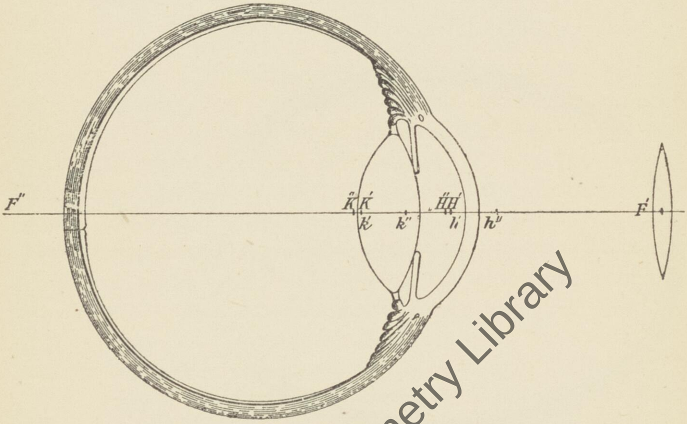

Fig. 19.

measurements two precautions must be taken, viz.: (1) The observer must keep his own accommodation absolutely relaxed; (2) The ophthalmoscope must be held in the correct position, 15·7 mm. from the cornea.

It has already been pointed out that the great advantage of putting the correcting lens in this place is that the retinal images are then exactly the same size as those of an emmetropic eye, and hence all the tests of visual acuteness are strictly comparable. The reason of this is clearly shown in *Figs.* 18 and 19. The capital letters above the axis show the positions of the unit and nodal points in an

emmetropic eye, the lower small letters show their position in the corrected ametropic eye. It will be observed that h' and k' occupy their old positions, but in the myopic eye (*Fig.* 18) h'' and k'' have been moved towards the retina by a distance that will be found to be exactly equal to F″Q, when, and only when, the correcting lens is placed in the anterior focal plane of the eye at F′.

This point is important, and its proof is given by using the dioptric formulæ, (2) *p.* 161 relating to a lens and an eye in which the final medium has an index $\mu = 1{\cdot}336$.

Suppose that a myopic eye requires for its correction a -6 D lens when placed at F′, i.e., when $t = \frac{1}{d_2}$. (*See Fig.* 18.)

Then $D' = d_1 + d_2 - td_1d_2$, or $-6 + 58{\cdot}636 - (-6) = 58{\cdot}636$D

$h' = -\frac{td_2}{D'} = -\frac{1}{D'} = -17{\cdot}054$ mm. or h' is at H′,

but $h'' = \frac{\mu t d_1}{D'} = \frac{1{\cdot}336\ (-6)\ ({\cdot}017054)}{58{\cdot}636} \backsimeq -{\cdot}00231$ m.

or $H''h'' = 2{\cdot}331$ mm. $= l$, for $l = (-{\cdot}3886)\ (-6) \backsimeq 2{\cdot}331$ mm.

As H″ and F″ have both been moved 2·331 mm. towards the retina so that F″ is now coincident with Q, k'' must also have moved to the same amount for k''Q must be equal to K″F″. In the hypermetropic eye (*Fig.* 19) by introducing a convex lens at F′ the points H″ and K″ have been moved away from the retina to h'' and k'' by the amount that the eye is too short. Hence the retinal image of any ametropic eye corrected in this way must be of exactly the same size as that of an emmetropic eye that views the same object from the same distance.

But correcting glasses are not usually worn at exactly this distance 15·706 mm. from the cornea or 17·054 mm. from the first unit point of the eye; they are usually worn at about 13·6 mm. from the cornea, i.e., 2·1 mm. nearer the cornea. The *p.r.* of the myopic eye we have been considering will then be 168·76 mm. from the lens instead of being 166·66 mm., and hence the lens must have

168·76 mm. for its focal length or be of a power − 5·9253 D, a trifle weaker than − 6 D, and with such a glass the retinal images will not be exactly of the same size as before, although, oddly enough, the two lenses at these respective distances are said to have the same ‘ effectivity ’. We shall deal later with the size of the retinal images when the correcting lens is not at F′.

Effectivity and Equivalence.—When the cardinal points of any complex optical system have been found, the result will always give an exactly *equivalent* simple system, which for small objects at any assigned distance will always form images of exactly the same size as those of the original system.

Now the power of a lens is fixed, but its effect on a wall behind it will vary with its distance from the wall. If the effect considered be only the formation of a definite image on the wall irrespective of its size, this effect will be produced, when the object is a distant one, by a + 10 D lens at 10 cm. from the wall or a + 16 D at 6·25 cm. Under these circumstances the two lenses are technically said to have the *same effectivity*. We are not here concerned with the propriety of the nomenclature, but with a clear explanation of this very curious special meaning of the term. Patients are much more anxious to see clearly, to have definite retinal images, than to insist on a specific size of the images, and for this reason the word ‘ effectivity ’ in this very special meaning has its uses.

Aphakia and Axial Length.—The same procedure is applied in cases of aphakia: $(F_a' - p)\,(F_a'' - q)$ or $f'l = F_a'F_a''$, and if these lengths are measured in millimetres,

$$\cdot 001\ \mathrm{D}' = \frac{1}{f'} = \frac{l}{F_a'F_a''} = \frac{l}{(23{\cdot}227)\,(-31{\cdot}031)} = \frac{l}{-720{\cdot}757}$$

$\therefore\ l = -\cdot 720757\ \mathrm{D}'$, and $\mathrm{D}' = -1{\cdot}38743\ l$, or as rough approximations $l \backsimeq -\cdot 72\,\mathrm{D}'$ and $\mathrm{D}' \backsimeq -1{\cdot}39\ l$.

Here D′ is the power of the lens which, placed at F_a' 23·277 mm. from the cornea, will correct the error l.

If we suppose that the eye was previously emmetropic its length AF″ will be 24·387 mm., but AF_a'' is 30·980 mm., so the aphakic eye may be regarded as 6·593 mm. too short; so $l = -6{\cdot}593$ mm.

$$D' = -1{\cdot}3874\ (-6{\cdot}593) = +9{\cdot}147\ \text{D}.$$

So a lens of this power placed at F_a' (nearly an inch from the cornea) would correct a previously emmetropic eye, presuming that no alteration in the curvature of the cornea had occurred after the operation. Clearly no one will ever wear a spectacle lens at this distance from his eye, and we must find later what glass when worn 13·6 mm. from the cornea will have the 'same effectivity' as + 9·147 D at 23·277 mm. from the cornea.

For estimating differences of level in an aphakic eye it is absolutely necessary to hold the ophthalmoscope at F_a', as totally erroneous conclusions will be drawn if held close to the eye.

For instance, if when the ophthalmoscope is held in the correct position (say, 23·3 mm. from the cornea) a retinal vessel is seen distinctly with + 14 D, while the edges of the disc are distinct with + 9 D, there must be a detachment of the retina, or a growth pushing it forwards to account for this difference of + 5 D.

$$l = -{\cdot}72 \times 5 = -3{\cdot}6\ \text{mm}.$$

The retina must, therefore, be raised 3·6 mm. If the previous formula were used for the complete eye, a difference of + 5 D would only imply a difference of level of 1·94 mm. This is a very important point, yet most books make no mention of it.

Correcting Lens in Normal Position.—It will presently be shown (*p.* 172) that if a lens D′ at a certain distance from the eye form a definite image on the retina, a lens D will also do so if it be placed n m. nearer the eye when $D = \dfrac{D'}{1 - nD'}$; if it be placed n m. further from the eye, n will be of course negative.

In the case of ametropia, as shown above, $l \approx - \cdot 39\ D'$ and $D' \approx - 2{\cdot}57\ l$ where D′ is the power of the correcting lens when placed at F′. If, however, the lens be placed 13·6 mm. from the cornea instead of 15·707 mm., i.e., say 2·1 mm. or ·0021 m. nearer the cornea, we must replace D′ or − 6 by $\frac{D'}{1 - nD'}$, thus $D = \frac{-\ 6}{1 + 6\ (\cdot 0021)} = \frac{-6}{1{\cdot}0126} = -\ 5{\cdot}9253\ D.$

Similarly if the value of D be given, and the value of l is required or $- \cdot 3886\ D'$ when the lens of power D′ is ·0021 m. further from the cornea than the lens D, $\frac{D}{1 + nD}$ or $\frac{D}{1 + \cdot 0021\ D}$ is substituted for D′. For rough approximations, unless the power of the lens be high these refinements need not be considered, and we may regard $D \approx D'$.

Aphakia.—It is very different in the case of aphakia, as the formula given ($\mathbf{D'} = -\ 1{\cdot}3874\ l$) gives the lens which is placed at 23·277 mm. from the cornea instead of 13·6 mm. from the cornea, and it is required to find **D** which is 9·677 mm. or ·009677 m. nearer the cornea. We found that in order to correct an aphakic emmetrope $D' = 9{\cdot}147$ D

$$\mathbf{D} = \frac{\mathbf{D'}}{1 - n\mathbf{D'}} = \frac{9{\cdot}147}{1 - \cdot 009677\ (9{\cdot}147)} = \frac{9{\cdot}147}{\cdot 9115} = 10{\cdot}035\ \text{D}.$$

This agrees very well with clinical results.

But it is often desirable to form an estimate of what the correction will be for an ametrope after the removal of his lens. This is rather a tedious investigation, and only a rough indication can be given as we are assuming that no change in the curvature of the cornea will have followed after the operation. The value is given of D which corrected the ametropia due to l' when placed 13·6 mm. away from the cornea (i.e., D′ if placed 15·7 mm. distant).

$$\text{Now } l' = -\ \cdot 3886\ D' \text{ or } \frac{-\ \cdot 3886\ D}{1 + \cdot 0021\ D}$$

$$D' = \frac{l + l'}{-\ \cdot 7208} = \frac{-\ 6{\cdot}593 - \dfrac{\cdot 3886\ D}{1 + \cdot 0021\ D}}{-\ \cdot 7208} = \frac{6{\cdot}593 + \cdot 40245\ D}{\cdot 7208 + \cdot 001514\ D}$$

But $\mathrm{D} = \dfrac{\mathrm{D}'}{1 - {\cdot}009677\,\mathrm{D}'}$

where

$$1 - {\cdot}009677\,\mathrm{D}' = \frac{{\cdot}7208 + {\cdot}001514\,\mathrm{D} - {\cdot}009677\,(6{\cdot}593 + {\cdot}40245\,\mathrm{D})}{{\cdot}7208 + {\cdot}001514\,\mathrm{D}}$$

$$= \frac{{\cdot}657 - {\cdot}00238\,\mathrm{D}}{{\cdot}7208 + {\cdot}001514\,\mathrm{D}}$$

$$\therefore\ \mathrm{D} = \frac{6{\cdot}593 + {\cdot}40245\,\mathrm{D}}{{\cdot}657 - {\cdot}00238\,\mathrm{D}} \approx \frac{6{\cdot}59 + {\cdot}4\,\mathrm{D}}{{\cdot}657 - {\cdot}0024\,\mathrm{D}}$$

As noted on *p.* 68 this formula is not reliable in high cases of myopia, but it will serve fairly well when the value of D is not very high.

Lenses of Same Effectivity.—Incident light from a distant source (i.e., with parallel rays) on encountering a convex lens of power D (focal length LF) in the plane at L will converge to the focus F. Similarly a convex lens of power D_1, focal length L_1F, if placed at L_1, will converge incident parallel rays to the same focus. The two lenses D and D_1 when in these positions are said to have the same *effectivity*, although of course they are not *equivalent*, for the images formed by them will be of different sizes, but they will occur in the same position. It has been previously shown how, by putting a certain lens in the anterior focal plane of a converging ametropic refracting system, one can alter the position of the image either forwards or backwards without altering its size; in such a case the whole system may then be made equivalent to the standard emmetropic system. It is very important to know what is the effectivity of a lens when placed in different positions, and in the case when the incident light consists of parallel rays the problem is delightfully simple.

In *Fig.* 20 clearly

$$L_1F = LF - LL_1 \text{ and } LF = L_1F + LL_1,$$

and if all these lengths are measured in metrical units—e.g., LL_1 is n metres in length—these expressions can be

given in the following dioptric form;

$$\frac{1}{D_1} = \frac{1}{D} - n, \text{ and } \frac{1}{D} = \frac{1}{D_1} + n;$$

$$\text{or } D_1 = \frac{D}{1 - nD}, \text{ and } D = \frac{D_1}{1 + nD_1}.$$

Further, if the symbol n be regarded as carrying inherently the sense of positive direction, i.e., in the direction of the transmission of light, as all the other expressions in the formula, $D_1 = \frac{D}{1 - nD}$ also carry inherently the same positive sign, this is the general vectorial formula which is universally true. If the positive lens is moved from L

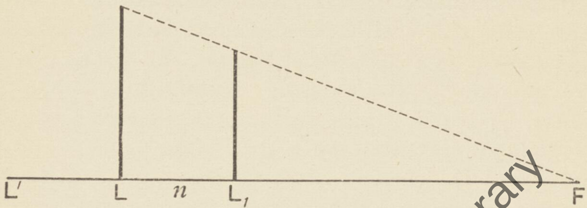

Fig. 20.

to L_1, nearer the eye, its effectivity will be diminished, as a stronger lens D_1 will be required in the situation L_1 to form an image at F. If the convex lens be removed away from the eye to a position L′, its effectivity will be increased, for a weaker lens at L′, namely D′, or $\frac{D}{1 + nD}$, will have the same effect as D at L. Note that the sign of n is now reversed, for LL′ is measured in the reverse direction to LL_1. For concave lenses the reverse results occur, for of course the sign of D is changed.

The following table shows the result of moving some lenses either towards the eye or away from the eye 9·5 mm. It is seen that if a + 5D lens corrects the eye in one position, it will require a + 5·25 D lens to produce the same effect if it be placed 9·5 mm. nearer the eye; whereas if

the lens is to be worn 9·5 mm. further from the eye, a lens of power + 4·77 D would have the same effectivity.

Position of Correcting Lens for Distance.

CONVEX.

D	D_1 $n = + \cdot 0095$ m.	D' $n = - \cdot 0095$ m.
+ 5	+ 5·25	+ 4·77
+ 10	+ 11·05	+ 9·13

CONCAVE.

D	D_1 $n = + \cdot 0095$ m.	D' $n = - \cdot 0095$ m.
− 5	− 4·77	− 5·25
− 10	− 9·13	− 11·05

When the glasses are used for near vision the matter is not quite so simple. Concave lenses when used either for near or distant objects have their effectivity increased when they are placed nearer the eye (i.e., a weaker lens in the new position will be as effective as the original lens in the old position) ; and their effectivity will be diminished when placed further from the eye.

Convex glasses are more troublesome ; the simplest way to explain the effect of alteration of their position is to realize that a convex lens is most effective when it is placed exactly midway between the object and its image, i.e., when the distance of the object is twice the focal length of the lens, or when $p = 2f'$. For instance, if a $+ 6\frac{2}{3}$D lens (focal length 15 cm.) be placed 30 cm. from the object, it is in the most effective position ; if the lens be placed 6·55 cm. either nearer to, or farther from the

object, it must be of power + 7D to produce the same effect. This note will serve for the present, but a fuller investigation will be required when we deal with the exact correction of astigmatism for near work (*pp.* 184–90).

The object of all pure mathematics is not to solve equations, useful though that art is, but to learn the correct procedure for the rigid analytical proof of a general law. Mathematical analysis is the only scientific method of discovering what is general and universal in a particular instance. Not even the elements of the method are given in any book that I have come across, and as some elementary knowledge of the subject is constantly required in optics, I venture to give this short outline of part of the subject. The above *Fig.* 20 is a very simple example of the method to pursue.

1. Make a diagram of any special instance considered.

2. Use purely geometrical reasoning. Pay no attention to the difference between positive and negative angles, let any expression such as PA denote simply a length without any sense of direction, so that PA = AP as in Euclid. Write down just what you obtain from the diagram. Then use any trigonometrical expression that may be needed, differentiate the expression if necessary, and transform it so that it gives the result (say, the angle ϕ) you wish to obtain. This formula will only apply to the special case considered in the diagram.

3. Now note the direction of the angles in the figure, those that are measured counter-clockwise are considered positive, while those in the clockwise direction are negative; similarly lengths in the direction that light travels are positive, those in the reverse direction are negative.

Rewrite the geometrical formula obtained so that each isolated term is denoted by expressions in the same direction. Thus, if, in the formula we have the expression AP, and in the figure AP is in the negative direction, we rewrite it as PA if we are putting all the expressions in the positive direction.

4. We then substitute the algebraic symbols which have a fixed direction for the geometrical expressions that have just been altered in accordance with rule (3). The resulting formula is universally true whatever value be given, positive or negative, to the symbols.

In the above case all the expressions, LF, LL_1, etc., are in the positive direction, so $D_1 = \dfrac{D}{1 - nD}$ is the general formula that is universally true. Other examples of not such a simple nature will be given later.

Back Focal Distance.—As in convex lenses the Second Unit Point H″ is not situated on the posterior surface of the lens (it may indeed be in front of the anterior surface), a great difficulty would arise in testing lenses by neutralization ; for a $-$ 16 D lens which is only 1 mm. thick (as in all concave trial lenses) would not neutralize a $+$ 16 D biconvex. The point H″ will be nearly in the centre of the lens, so the point F″ will be nearly half the thickness of the lens nearer its posterior surface than it should be for complete neutralization. For this reason concave lenses are numbered according to their true value, and convex lenses are numbered so that they will be neutralized by concaves of the same numerical value. Hence a lens from the trial case numbered $+$ 20 D will probably really be only about $+$ 18·75 D according to its thickness. For this reason, though the *back focal distance* of the trial lens of $+$ 16 D in the experiment on *p.* 162 should be 6·25 cm., the power of the lens will really be less than $+$ 16 D, and so the size of the images will not be exactly the same. Practically it is much more convenient to use convex lenses which are numbered as if their focal distance were measured from their posterior surface, which is placed at the assigned distance (13·6 mm.) from the cornea, and this is now the universal custom. If the thickness of a convex lens be known, and the dioptric value of each of its surfaces be also known, its *back focal distance* can be found. This or its reciprocal, the effective dioptric power (D_b) as measured from the posterior surface of the lens, is always required in prescribing periscopic lenses.

Thickness of the Lens.—The thickness (t) of a convex lens necessarily depends upon its size and upon its curvature. A periscopic lens will be of somewhat large size, say 40 mm. in its greater diameter (an 00 eye as it is called) ; if then a disc be ground to a convexity of $+$ 10 D (i.e., a curvature of radius 52·3 mm. when $\mu = 1·523$) until the edge be infinitely thin, its axial thickness will be $r\ (1 - \cos\theta)$. But we do not know $\cos\theta$, we only know

sin θ which is $\frac{p}{r}$, where p is half the diameter of the lens or 20 mm.

So $t = r - r \cos\theta = r - r\sqrt{1 - \sin^2\theta} = r - \sqrt{r^2 - p^2}$

or $t = 52{\cdot}3 - \sqrt{2735{\cdot}29 - 400} = 52{\cdot}3 - 48{\cdot}32 = 3{\cdot}98$ mm.

As this planoconvex lens must be at least 1 mm. thick at the edge, we add 1 mm. to this value and obtain 4·98 mm. as the minimum thickness of this + 10 D lens. It is easily seen that, whatever the curvature of its surfaces, the axial thickness must be that of a planoconvex lens of the same size and power.

Practically a fairly good estimate of the thickness of a lens of any power D, the greatest diameter of which is 40 mm., can be obtained by using the empirical rule :—

$$t = 1 + {\cdot}4\ \mathrm{D} = 5 \text{ mm., if } \mathrm{D} = 10.$$

Now in the case of a meniscus, such as is required for a + 10 D periscopic lens for distance, the ocular curvature should be that given by a − 7 D tool, what must be the anterior curvature that will have the same effectivity as the lens which is numbered + 10 D in the trial case? Clearly we cannot use formula (3) *p.* 161, for though this gives a lens of focal length exactly 10 cm. it will be found that the Second Unit Point is in front of the anterior surface, and so the effectivity is too great. On *p.* 73 it was stated that patients complain of a curvature of the field when the periscopic lens is too convex, so it is very important to avoid this error.

If B denote the apex of the concave ocular surface, F″B denotes the back focal distance of the lens, and $\mathrm{F''B} = \frac{-1}{\mathrm{D_b}}$ where $\mathrm{D_b}$ is the effective (back) dioptric power of the lens, which is sometimes loosely called its " vertex " refraction.

$$\text{Now } \mathrm{F''B} = \mathrm{F''H''} + \mathrm{H''B} = \frac{f_2''\,(f_1'' + t)}{t + f_1'' - f_2'}$$

$$\therefore -\mathrm{D_b} = \frac{1}{f_2''} - \frac{f_2'}{f_2''\,(f_1'' + t)} = -\,\mathrm{d_2} + \frac{\mu}{t + f_1''}$$

$$\therefore \mathrm{D_b} - \mathrm{d_2} = \frac{-\mu}{t + f_1''} = \frac{-\,\mu\,\mathrm{d_1}}{t\mathrm{d_1} - \mu} = \frac{\mu\,\mathrm{d_1}}{\mu - t\mathrm{d_1}}$$

$$\therefore \frac{1}{D_b - d_2} = \frac{1}{d_1} - \frac{t}{\mu} \text{ or } d_1 = \frac{\mu(D_b - d_2)}{\mu + (D_b - d_2)t}$$

In the case considered $D_b - d_2 = 10 + 7$
and $t = \cdot 00498$ m.

$$d_1 = \frac{17(1 \cdot 523)}{1 \cdot 523 + 17(\cdot 00498)} = \frac{25 \cdot 891}{1 \cdot 60766} \backsimeq 16 \cdot 1$$

Size of the Retinal Image.—As the distance of the object, whether 33 cm. or 25 cm., makes a considerable difference in the size of the retinal image, it is well to take a very distant object as our standard example. If the moon be the object, it may be said to subtend at the eye an angle of 31′, whatever be its distance, though that varies to some extent. On referring to *Fig.* 17 it is seen that $\frac{ab}{bK''} = \frac{AB}{BK'}$ = tan BK′A, but as the image will now be formed in the second focal plane of the eye, we must substitute F″ for b, and tan 31′, or ·009, for tan BK′A, so aF″ = F″K″ tan 31′. In the table, *p.* 155

$$K''F'' = F'H' = 17 \cdot 054,$$

consequently i or aF″ = − 17·054(·009) = − ·1535 mm. This will be the diameter of the retinal image of a full moon in an emmetropic eye. If the eye be ametropic, and if the correcting lens be situated in the anterior focal plane of the eye, as previously shown, the image will be of exactly the same size.

If the correcting lens be + 5 D when worn in the first focal plane of the eye, 17·054 mm. from H′, the similarly effective lens will be + 5·0532 D when worn 2·106 mm. nearer the eye, i.e. 14·948 mm. from H′ or 13·6 mm. from the cornea.

$$\text{For } D' = \frac{D}{1 - \cdot 002106\,D} = \frac{5}{1 - \cdot 01053} = \frac{5}{\cdot 98947} = 5 \cdot 0532.$$

The image (i') will now be − ·009 **F′** or $\frac{-\cdot 009}{(\cdot 001)\mathbf{D}}$ and we must find **F′** or **D** for the whole system.

The reason why the factor ·001 was used in conjunction with D was because the result was required in millimetres; without that factor the result would be given in metres.

$$D = 5{\cdot}0532 + 58{\cdot}636 - ({\cdot}014948)\,(5{\cdot}0532)\,(58{\cdot}636)$$
$$= 59{\cdot}2601.$$

$$i' = \frac{-\,{\cdot}009}{({\cdot}001)\,D} = \frac{-\,9}{D} = \frac{-\,9}{59{\cdot}26} = -\,{\cdot}15187 \text{ mm.}$$

Clearly the size of the image varies as the first focal length F' of the eye or of the system, or inversely as the power D,

$$\text{so } \frac{i'}{i} = \frac{F'}{F'} = \frac{D}{D}.$$

In this case $\frac{i'}{i} = \frac{58{\cdot}637}{59{\cdot}261} = {\cdot}9895.$

The above has been worked out with needless detail and accuracy to show the actual procedure that should be adopted.

If the case of $-\,6$ D myopia (*p.* 168) be considered, when the $-\,6$ D lens is placed in the first focal plane, i will be equal to that of an emmetrope, but when the similarly effective $-\,5{\cdot}925$ D is placed 2·106 mm. nearer the cornea

$$D = -\,5{\cdot}925 + 58{\cdot}636 + ({\cdot}014948)\,(5{\cdot}925)\,(58{\cdot}636)$$
$$= 57{\cdot}904.$$

$$\therefore \frac{i'}{i} = \frac{58{\cdot}636}{57{\cdot}904} = 1{\cdot}0126.$$

When a concave lens is advanced towards the cornea there is a slight magnification, but when a convex lens is advanced there is a slight diminution; in both these cases, however, the alteration in size is practically negligible.

Image in Corrected Aphakia.—On *p.* 170 it was found that $+\,10{\cdot}085$ D, if placed 13·6 mm. from the cornea, should correct an aphakic who had previously been been emmetropic; we now wish to find the size of his retinal image in comparison with that of an emmetrope. The lens will now be 13·55 mm. from H'^{a}, so

$$D = 10{\cdot}035 + 43{\cdot}053 - ({\cdot}01355)\,(10{\cdot}035)\,(43{\cdot}053)$$
$$= 47{\cdot}234\ \text{D}.$$

$$\text{and } \frac{i'}{i} = \frac{\text{D}}{\text{D}} = \frac{58{\cdot}636}{47{\cdot}234} = 1{\cdot}2414.$$

Consequently a corrected aphakic sees an object 24 per cent larger than an emmetrope ; test type results will not be comparable with those in ametropia ; for instance, an aphakic who sees $\frac{6}{18}$ has really a visual acuteness very little more than what would be denoted by $\frac{6}{24}$ in an emmetrope or a corrected ametrope. Hence it is a grave mistake to remove a cataractous lens for *visual* purposes if the other eye has good vision. On one day I once saw three patients with cataracts removed from one eye while the vision in the other eye was good. They all had suffered from extreme discomfort from their inability to fuse the retinal images of such a different size ; two of them were wearing shades over their aphakic eyes, and the remaining one I presented with a shade to follow their example.

It will be of interest to see where the Second Unit Point is situated in the corrected aphakic just considered.

$$h'' \text{ or } \text{H}''\text{H}_a'' = \frac{\mu\, d_1\, t}{\text{D}} = \frac{1{\cdot}336\ (10{\cdot}035)\ ({\cdot}01355)}{47{\cdot}234} = {\cdot}003846 \text{ m.}$$

but $\text{H}_a''\text{A}_0 = {\cdot}051$ mm. $\therefore$ $\text{H}''\text{A} = 3{\cdot}897$ mm.

while $\text{H}''\text{F}'' = \frac{\mu}{\text{D}} = 28{\cdot}2847$, so $\text{A F}'' = 24{\cdot}3877$

and we see from the table (*p.* 155) that in an emmetropic eye $\text{A}_0\text{F}'' = 24{\cdot}387$ mm. so that our result is correct to five figures with an error of less than ·001 mm.

It is an interesting little problem to find the total power **D** of the system if the same lens (+ 10·035 D) be withdrawn 5 mm. further from the eye, so that $t = {\cdot}01855$ m. from H_a' ; **D** will be 45·074, distinctly less than before, but as h'' now = 5·517 mm.

q or $\text{F}''\text{H}'' = -24{\cdot}387 - {\cdot}051 - 5{\cdot}517 = -29{\cdot}955$ mm. The distance of the object for which the system will now be adjusted will be a little more than 2·1 m. The value of q is the distance of the Second Unit Point of the system from the retina, and

$$p = \frac{qf'}{q - f''} \text{ or } \frac{q}{\mu + q\mathrm{D}}$$

$$\text{i.e. } \frac{-\cdot 029955}{1\cdot 336 - (\cdot 029955)\ (45\cdot 074)} = \frac{-\cdot 029955}{-\cdot 014}$$

$$p \backsimeq 2\cdot 1 \text{ metres}$$

and the size of the image (i') as compared with that of an emmetropic eye is roughly $\frac{i'}{i} = \frac{58\cdot 637}{45\cdot 074} \backsimeq 1\cdot 301$.
This explains why aphakics remove their spectacles further from their eyes in order to see objects such as pictures in the middle distance.

Tilted Lens.—If a spherical lens be so inclined that the plane of the glass makes an angle with the incident wave-front of light, the refracted pencil is astigmatic, so that when the incident pencil is centric it is refracted as though through a spherocylinder. As the pupil allows only a very narrow pencil to enter the eye, this property of tilted lenses is often of service to those patients who cannot afford spherocylinders. The formula for oblique *centric* pencils through a lens is the following:

$$\frac{\cos^2\phi}{v_1} - \frac{\cos^2\phi}{u} = \frac{1}{v_2} - \frac{1}{u} = \frac{\sin(\phi - \phi')}{\sin\phi'}\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

where u, v_1, and v_2 represent the distances of the Unit Points of the lens from the source of light, the first focal line, and the second focal line. When the correction is to be used for distance, $u = \infty$, and this formula suffices:

$$\frac{\cos^2\phi}{v_1} = \frac{1}{v_2} = \frac{\sin(\phi - \phi')}{\sin\ '}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

When u, v_1, v_2, r_1, and r_2, are measured in millimetres

$$\frac{1}{v_1} = -\cdot 001\ \mathrm{D}_1, \qquad \frac{1}{v_2} = -\cdot 001\ \mathrm{D}_2,$$

and since $\frac{\quad}{\mu - 1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{f''} = -\cdot 001\mathrm{D}$

$\frac{1}{r_1} - \frac{1}{r_2} = \frac{-\cdot 001\ \mathrm{D}}{\mu - 1}$ and the expression in

dioptres will be $\cos^2\phi \cdot D_1 = D_2 = \frac{\sin(\phi - \phi')}{\sin \phi'} \quad \frac{D}{\mu - 1}$

where D_1 and D_2 represent the power in the two meridians of the lens D tilted the angle ϕ round the axis of its least refraction. Since this is equivalent to a spherical glass of dioptric strength D_2 combined with a cylindrical lens of dioptric strength D_c,

$$D_c = D_1 - D_2 = D_2\left(\frac{1 - \cos^2\phi}{\cos^2\phi}\right) = D_2 \tan^2\phi.$$

We have then $D_2 = \frac{\sin(\phi - \phi')}{\sin \phi'} \cdot \frac{D}{\mu - 1}$, and $D_c = D_2 \tan^2 \phi$, which are the simplest forms for calculation.

For instance, the value of D_2 in the case of a + 10 D lens inclined at an angle of 30° can be calculated or found from the table on *p.* 69 to be 10·948. Now $\tan 30° = \frac{1}{\sqrt{3}}$

$$\therefore \tan^2 30 = \frac{1}{3}, \text{ so } D_c = \tfrac{1}{3}\ (10{\cdot}948) = +\ 3{\cdot}649.$$

Practically this method can only be used when the spherical component is rather high, and when the axis of the cylinder is horizontal, so that it is of service only in some cases of high myopia, and in many cases of aphakia.

The Ophthalmometer.—On *p.* 84 a brief description of the ophthalmometer was given, and it was shown that the reading of the instrument only gave the concave cylinder which, when placed in contact with the cornea, would correct the astigmatism that was due to the anterior surface of the cornea. This cylinder will not correct even this astigmatism when it is worn 13·6 mm. in front of the cornea. The note on lenses of the same effectivity (*p.* 171) will enable us to find the formula for drawing up the tables for the correcting cylinder when placed 13·6 mm. from the cornea (*p.* 183).

Let L_1 (*Fig.* 20) represent the position of the corneal plane, and L that of the spectacle plane, and let D_1 when

placed at L be the correction for the meridian that requires the stronger convex (or the weaker concave) lens, and let D_2 be the weaker convex (or the stronger concave) lens for the other meridian at right angles to the first. Thus if + 4D sph. + 2 D cyl. ax. 90° be the true correction at L, $D_1 = +6$ ax. 180°, and $D_2 = +4$ ax. 90° and the concave cylinder required is

$$A = D_2 - D_1 = +4 - 6 = -2.$$

Similarly let $A' = D_2' - D_1'$ for the contact correction at L_1.

The reading of the instrument gives us A′, and we can find D_1 by the fogging method from the astigmatic chart (or by retinoscopy); we must find A in terms of D_1, A′ and n which we are taking to be ·0136 m.

From (1) *p.* 172 we have $D = \dfrac{D'}{1 + nD'}$, and $D' = \dfrac{D}{1 - nD}$

$$\therefore \frac{D'}{D} = 1 + nD' = \frac{1}{1 - nD}$$

$$\text{Now } A = D_2 - D_1 = \frac{D_2'}{1 + nD_2'} - D_1 = \frac{D_1' + A'}{1 + n(D_1' + A')} - D_1$$

$$\therefore A = \frac{D_1' + A' - D_1(1 + nD_1') - nA'D_1}{1 + nD_1' + nA'}$$

$$= \frac{D_1' + A' - D_1' - nA'D_1}{1 + nD_1' + nA'}$$

$$\therefore A = \frac{A'(1 - nD_1)}{\dfrac{1}{1 - nD_1} + nA'} = \frac{A'(1 - nD_1)^2}{1 + nA'(1 - nD_1)}$$

Suppose that $D_1 = +10$, and A′ is found by the instrument to be − 5 and the axis horizontal; then $1 - nD = 1 - {\cdot}136 = {\cdot}864$

$$A = \frac{A'({\cdot}864)^2}{1 + {\cdot}0136A'({\cdot}864)} = \frac{-3{\cdot}73248}{1 - {\cdot}068({\cdot}864)} \approx -3{\cdot}965 \approx -4.$$

and the correction would be + 10 D sph. − 4 D cyl. ax. 0°; or + 6 D sph. + 4 D cyl. ax. 90°.

If D_1 were -10, $1 - nD = 1{\cdot}136$, and this number must replace ·864 in the above expression. It is a tedious calculation even with logarithms, and I should be very grateful if any reader would point out to me any error he may find in my tables. They are given in detail below for those who habitually use this method.

I personally hardly ever use the instrument for estimating the astigmatic correction, as much more reliance can be placed on retinoscopy, but without such a table or the use of the above formula, hopelessly erroneous conclusions may be drawn.

CONVEX.

D	Ophthalmometric Reading.							
	— 1	— 2	— 3	— 4	— 5	— 6	— 7	— 8
+ 0	— 1·014	— 2·056	— 3·128	— 4·230	— 5·365	— 6·533	— 7·737	— 8·977
+ 1	— ·986	— 2·000	— 3·041	— 4·113	— 5·215	— 6·349	— 7·517	— 8·720
+ 2	— ·959	— 1·944	— 2·956	— 3·997	— 5·067	— 6·168	— 7·301	— 8·467
+ 3	— ·932	— 1·889	— 2·872	— 3·883	— 4·921	— 5·989	— 7·088	— 8·218
+ 4	— ·906	— 1·836	— 2·790	— 3·771	— 4·778	— 5·813	— 6·878	— 7·974
+ 5	— ·880	— 1·782	— 2·709	— 3·660	— 4·637	— 5·641	— 6·672	— 7·733
+ 6	— ·854	— 1·730	— 2·629	— 3·551	— 4·498	— 5·471	— 6·470	— 7·497
+ 8	— ·804	— 1·628	— 2·473	— 3·339	— 4·227	— 5·139	— 6·075	— 7·036
+ 10	— ·755	— 1·529	— 2·321	— 3·133	— 3·965	— 4·819	— 5·693	— 6·592
+ 12	— ·708	— 1·433	— 2·175	— 2·935	— 3·712	— 4·502	— 5·326	— 6·163
+ 14	— ·663	— 1·340	— 2·034	— 2·743	— 3·468	— 4·211	— 4·971	— 5·750
+ 16	— ·619	— 1·251	— 1·897	— 2·557	— 3·233	— 3·924	— 4·630	— 5·353
+ 18	— ·576	— 1·165	— 1·765	— 2·379	— 3·006	— 3·647	— 4·301	— 4·971
+ 20	— ·535	— 1·081	— 1·639	— 2·207	— 2·788	— 3·381	— 3·986	— 4·605

Note that D denotes the highest convex (or the lowest concave) power required. Thus if with + 8 D the reading is — 5 D astigmatism, axis horizontal, it is seen that the cylinder required is — 4·227 D, so + 8 sph. — 4·25 cyl. axis horizontal, i.e., + 3·75 sph. + 4·25 cyl. ax. vertical would be the prescription.

CONCAVE.

D	Ophthalmometric Reading.							
	− 1	− 2	− 3	− 4	− 5	− 6	− 7	− 8
− 0	− 1·014	− 2·056	− 3·128	− 4·230	− 5·365	− 6·533	− 7·737	− 8·977
− 1	− 1·042	− 2·113	− 3·215	− 4·349	− 5·517	− 6·720	− 7·960	− 9·238
− 2	− 1·070	− 2·171	− 3·304	− 4·470	− 5·672	− 6·910	− 8·186	− 9·503
− 3	− 1·099	− 2·230	− 3·394	− 4·593	− 5·829	− 7·103	− 8·417	− 9·773
− 4	− 1·128	− 2·289	− 3·485	− 4·718	− 5·988	− 7·299	− 8·651	− 10·047
− 5	− 1·157	− 2·349	− 3·578	− 4·844	− 6·150	− 7·497	− 8·888	− 10·325
− 6	− 1·187	− 2·411	− 3·672	− 4·972	− 6·314	− 7·699	− 9·129	− 10·607
− 8	− 1·248	− 2·535	− 3·863	− 5·233	− 6·647	− 8·111	− 9·622	− 11·185
− 10	− 1·311	− 2·663	− 4·060	− 5·502	− 6·993	− 8·534	− 10·129	− 11·780
− 12	− 1·375	− 2·794	− 4·261	− 5·777	− 7·346	− 8·970	− 10·651	− 12·393
− 14	− 1·440	− 2·929	− 4·468	− 6·061	− 7·709	− 9·417	− 11·187	− 13·023
− 16	− 1·507	− 3·067	− 4·680	− 6·351	− 8·082	− 9·877	− 11·738	− 13·671
− 18	− 1·576	− 3·208	− 4·897	− 6·648	− 8·464	− 10·349	− 12·305	− 14·338
− 20	− 1·646	− 3·352	− 5·120	− 6·953	− 8·856	− 10·832	− 12·886	− 15·023

Lenses of Same Effectivity for Near Work.—We must now consider the problem of *p.* 170 when a near object is viewed. An emmetrope must use $3\frac{1}{3}$ D of accommodation to see an object distinctly at 30 cm. from his spectacle-plane, for we have agreed to denote the amount of the accommodation by the power of the convex lens placed in the spectacle-plane that will be similarly effective. Now it is usually considered that a hypermetrope of + 4 D or + 8 D when wearing his distance correction will also require exactly $3\frac{1}{3}$ D of accommodation to read at this distance (30 cm.). But this is not the case, for it will be found that both convex and concave lenses are less effective in correcting refractive errors when used for near work than when used for distance (*p.* 64). It is a point that requires careful consideration, as so far it has not been mentioned in any of the books, although I have recently found that the late S. D. Chalmers read

a paper on the subject before the Optical Society in 1906.

Let P denote the position of the object in front of the convex lens at L, and let L_1 denote the position of the two Unit Points of the eye; these for practical purposes we may regard as coincident in the situation of the cornea of the Simplified Schematic Eye (*p.* 157); then LL_1 or $n = 13{\cdot}6$ mm. or $\cdot 0136$ m. while PL or $p = 30$ cm. or $\cdot 3$ m. Let Q be the image of P formed by the lens of power D at L or by the lens D_1 of the same effectivity placed at L_1, i.e., a lens in contact with the cornea.

We have $PL_1 = p + n$ which we may denote by p_1 and $QL = q$, so $QL_1 = QL + LL_1 = q + n$ or q_1

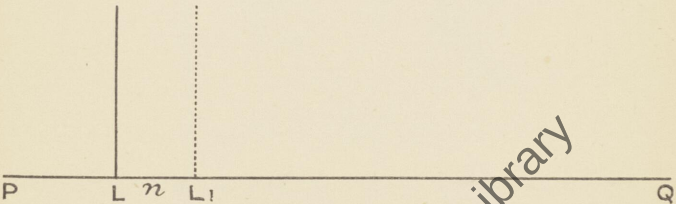

Fig. 21.

Now $D = \dfrac{q-p}{pq}$, and $D_1 = \dfrac{q+n-(p+n)}{(p+n)(q+n)} = \dfrac{q-p}{p_1(q+n)}$

$$\therefore D_1 p_1 (q+n) = Dpq \text{ and as } q = \frac{p}{1-pD}$$

$$\frac{D_1 p_1 (p + n - npD)}{1-pD} = \frac{Dp^2}{1-pD}$$

$$\therefore D_1 \ (p_1^2 - npp_1 D) = Dp^2 \qquad \ldots\ldots\ldots\ldots\ldots \ (1)$$

$$\text{or } D_1 = \frac{Dp^2}{p_1^2 - npp_1 D} = \frac{p^2}{\frac{p_1^2}{D} - npp_1}$$

(It will be noted that all the symbols in the formula have positive values, and are so represented in the diagram, therefore the formula is universally true whatever changes may be made in the signs of any symbol used.)

On replacing the symbols by their values in metres, $p = \cdot 3$, $n = \cdot 0136$, $p_1 = \cdot 3136$, we obtain

$$D_1 = \frac{\cdot 09}{\dfrac{\cdot 0983449}{D} - \cdot 00127949}$$

In this way the value of D_1 has been obtained in the following little table :—

CONVEX.

D_0 $p = \infty$	${}_1D_0$ $p = \infty$	D_1 $p = \cdot 3$	D' $p = \cdot 3$
+ 3	+ 3·128	+ 2·857	+ 3·272
+ 4	+ 4·230	+ 3·862	+ 4·360
+ 7	+ 7·737	+ 7·048	+ 7·616
+ 8	+ 8·877	+ 8·172	+ 8·699

CONCAVE.

D_0 $p = \infty$	${}_1D_0$ $p = \infty$	D_1 $p = \cdot 3$	D' $p = \cdot 3$
− 4	− 3·794	− 3·480	− 4·382
− 8	− 7·215	− 6·631	− 8·785

The column headed ${}_1D_0$ denotes the similarly effective corneal contact lens, when the lens D_0 placed at L is used for distance, and may be regarded as indicating the real static refractive error of the eye. The static refraction of the eye is always determined by that lens which enables the patient to see distinctly at a distance when the accommodation is completely relaxed, whether the subjective method or that of retinoscopy be employed. The values in the column ${}_1D_0$ are obtained by the formula (*p.* 172) $D_1 = \frac{D}{1 - nD}$. Neglect for the moment the column headed D'.

Now the values in the column D_1 are in all cases less than the values in the column ${}_1D_0$, i.e., a lens D_0 which corrects the refractive error for distance will under-correct this static error if used for close work. In simple myopia this will be rather an advantage, but in high hypermetropia one should be ready to give a presbyopic addition for close work at an earlier age than usual. In every case of high astigmatism attention should be directed to this point.

Suppose a patient complains of morning headache, and some discomfort in reading which has lately occupied most of his time. One suspects subnormal accommodation, but his accommodation is above the mean for his age, and his glasses, which he wears constantly, are found to be correct + 4 D cyl. for each eye. On looking at the table one will see that his real corneal astigmatism is + 4·23 D, but when he uses these + 4 D cylinders for reading they no longer correct the actual corneal astigmatism but only 3·862, so there remains ·368 D of astigmatism uncorrected. Clearly he requires a higher cylinder for reading, and all his trouble is explained by the elementary laws of optics.

Of course the spherical correction for reading demands a slight increase, but this is a mistake that is not likely to arise, as the reading correction should only be given after trial. It will be seen that what I previously stated (*p.* 14) was not accurate—a corrected ametrope cannot be regarded in every respect as an emmetrope—but the introduction of these refinements at that stage would only confuse the reader.

It will be noticed that D_1 is less than D_0 when D_0 is + 3 or + 4, but greater than D_0 when it is + 8. When $D_0 = 6{\cdot}5221$, $D_1 = D_0$ for then if $p = {\cdot}3$, $q = -{\cdot}3136$, and of course if $p_1 = {\cdot}3136$, $q_1 = -{\cdot}3$, for then $D_1 p_1 (q + m) = D\,pq$. This explains the difficulty that a careful critical reader will discover.

It is clear, then, that whenever distance lenses are used for close work the refractive error will be under-corrected, but if an additional convex lens be given for reading that will entirely remove any exertion of the accommodation

(as in aphakic cases), no addition will be required in the cylinder which corrects his astigmatism for distance. The reason, of course, is that the convex additional lens may be regarded as forming a virtual image of the object at a great (infinite) distance. But it will be well to work out our previous case with + 4 D of astigmatism if + 3 D were provided for reading. As the object is presumed to be at a distance of 30 cm. the additional lens should be $3\frac{1}{3}$ D, but we will use the values that we have already tabulated for + 3 D.

The corneal astigmatism is + 4·23 D and the D_1 value of + 3 D is + 2·857, therefore the total D_1 value of the sum of the two is 7·087 D. What must be the value of the correcting lens to give this value?

From (1) $D\ (p^2 + npp_1D_1) = D_1\ p_1^2$

$$D = \frac{p_1^2}{\frac{p^2}{D_1} + npp_1} = \frac{\cdot 0983449}{\frac{\cdot 09}{D_1} + \cdot 00127949}$$

As $\frac{\cdot 09}{7\cdot 087} \approx \cdot 0127$, $D \approx \frac{\cdot 098345}{\cdot 0139795}$ which is less than 7·035,

so the correction may be taken as + 3 D sph. + 4 D cyl. even when the spherical lens is not quite strong enough.

But for practical purposes one must devise some simpler rule than these troublesome calculations. We first must find what lens (D′), when used for near work (at 30 cm.), will have the same effect (as a corneal contact-lens), as the lens D_0 has when used at a distance. We have merely to substitute $\frac{D_0}{1 - pD_0}$ for D_1 in the last equation.

$\frac{p^2}{D_1} + npp_1$ becomes $\frac{p_2 - npD_0\,(p - p_1)}{D_0} = \frac{p^2}{D_0} + n^2p.$

$$\therefore\ D' = \frac{p_1^2}{\frac{p^2}{D_0} + n^2p} = \frac{\cdot 0983449}{\frac{\cdot 09}{D_0} + \cdot 000055488}$$

By means of this formula I have given the value (D′) which, when used for close work (30 cm.), will have the same effect in correcting refractive errors as D_0 has, when used for distance. I have also given the corneal contact-lens ($_1D_0$) which corresponds to both D_0 and D′.

LENSES OF SAME EFFECTIVITY FOR NEAR AND FOR DISTANT WORK.

Convex.			Concave.		
D_0 $p = \infty$	$_1D_0$ $p = \infty$	D′ $p = \cdot 3$	D_0 $p = \infty$	$_1D_0$ $p = \infty$	D′ $p = \cdot 3$
+ 1	+ 1·014	+ 1·092	— 1	— ·987	— 1·093
+ 2	+ 2·056	+ 2·183	— 2	— 1·947	— 2·188
+ 3	+ 3·128	+ 3·272	— 3	— 2·882	— 3·284
+ 4	+ 4·230	+ 4·360	— 4	— 3·794	— 4·382
+ 5	+ 5·365	+ 5·447	— 5	— 4·682	— 5·481
+ 6	+ 6·533	+ 6·532	— 6	— 5·549	— 6·581
+ 7	+ 7·737	+ 7·616	— 7	— 6·392	— 7·682
+ 8	+ 8·877	+ 8·699	— 8	— 7·215	— 8·785
+ 10	+ 11·574	+ 10·860	— 10	— 8·803	— 10·995
+ 12	+ 14·340	+ 13·016	— 12	— 10·316	— 13·210
+ 14	+ 17·292	+ 15·167	— 14	— 11·761	— 15·431
+ 16	+ 20·450	+ 17·313	— 16	— 13·141	— 17·658
+ 18	+ 23·835	+ 19·453	— 18	— 14·460	— 19·890
+ 20	+ 27·473	+ 21·588	— 20	— 15·723	— 22·127

On looking at the values of D′ as compared with D_0 it is seen that up to about 7 D they are about 9 per cent greater, e.g., if $D_0 = + 5$, $D' = + 5{\cdot}447$; but if $D_0 = - 5$, $D' = - 5{\cdot}481$, so that both may be regarded as 5·45. We have found that when + 3 D is added to a reading lens the power of the cylinder is unaltered, and we may say that if the reading addition be + 2 D, the cylinder should be increased by 3 per cent, if it be only

+ 1 D, the cylinder should be increased by 6 per cent; and if no spherical addition be made for reading purposes, the cylinder should be increased by 9 per cent. This is very important when the cylinder required is high; a — 10 D cyl. is not unusual in conical cornea, if there is no spherical component, it is seen from the table that — 11 D cyl. would be required for close work.

Should the patient see best at a distance with — 10 D sph. — 10 D cyl., the combination for reading would be — 11 D sph. — 11 D cyl. If he were given for reading the same spherical lens (— 10 D), it should be regarded as providing him with + 1 D for the relief of his accommodation, and so his cylinder should only be increased about 6 per cent.

A study of the increase of the values of the higher values of the ${}_1D_0$ columns will explain the extraordinary ophthalmometric tables on *p.* 183–4. For instance, the difference between + 20 and + 16 is 4, but the difference between the corresponding ${}_1D_0$ values is over 7. Consequently, if we examined with the ophthalmometer an eye requiring + 20 D — 4 D cyl. the ophthalmometric reading would be — 7 D of astigmatism. Similarly the difference between — 14 and — 20 is 6, but the difference between the corresponding values of ${}_1D_0$ is less than 4. Hence we are not surprised that a patient who wears — 14 D — 6 D cyl. should give a reading of only — 4 D of astigmatism on the ophthalmometer.

Power of an Oblique Cylinder.—A cylindrical lens of power + 4 D only exerts that power at right angles to its axis; it exerts no power in the plane of its axis, and in the intermediate planes it exerts powers intermediate between 0 and 4. Suppose, for instance, a patient requires + 3 D sph. + 4 D cyl. ax. 120°, and we wish to combine with this lens a prism of 3 ▽ base in, and one of 1 ▽ base up. It can be done quite easily by decentration,

but we must know what is the power of this sphero-cylinder in the vertical meridian, and what is its power in the horizontal meridian.

To find the power (C_θ) of a cylinder C in the horizontal meridian when its axis is placed at the angle θ.

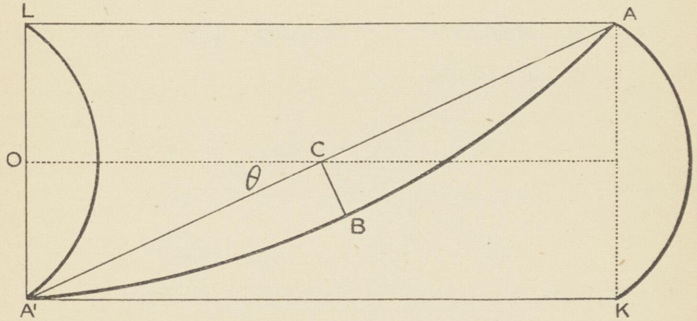

Fig. 22.

Let ALA′K represent half a cylinder with its convex surface anterior, and let ACA′B be the surface exposed on making a section through this plane at the angle θ with the median plane OC. Any oblique section of a cylinder will have an elliptical contour, and we must find the radius of curvature of the ellipse ABC at the point B.

In the semi-ellipse A′BA, $a = $ CA and $b = \text{CB} = \text{OL} = r$ the radius of the cylinder.

$$\sin\,\theta = \frac{\text{A}'\text{O}}{\text{A}'\text{C}} = \frac{\text{OL}}{\text{CA}} = \frac{b}{a} \qquad\qquad \therefore\quad a = \frac{b}{\sin\theta}$$

In an ellipse the position of any point P can be determined by the expression $x = a\,\cos\,\phi$ and $y = b\,\sin\,\phi$ where ϕ is the eccentric angle of the point P, and if r' represents the radius of curvature at the point P, those who have not forgotten their school mathematics will remember that:—

$$r' = \frac{\left\{\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2\right\}^{\frac{3}{2}}}{\frac{dx}{d\phi}\frac{d^2y}{d\phi^2} - \frac{dy}{d\phi}\frac{d^2x}{d\phi^2}} = \frac{(a^2 \sin^2\phi + b^2 \cos^2\phi)^{\frac{3}{2}}}{ab\,(\sin^2\phi + \cos^2\phi)}$$

$$= \frac{(a^2 \sin^2\phi + b^2 \cos^2\phi)^{\frac{3}{2}}}{ab}$$

At the point B where the eccentric angle $\phi = 90°$ this last expression reduces to $r' = \frac{a^3}{ab}$ or $\frac{a^2}{b} = \frac{b^2}{b \sin^2\theta}$

$$= \frac{b}{\sin^2\theta} \quad = \frac{r}{\sin^2\theta}$$

Now C_θ in dioptres $= \frac{1}{f'_\theta} = \frac{\mu - 1}{-r'} = \frac{\mu - 1}{-r}\sin^2\theta,$

$$\text{and } C = \frac{\mu - 1}{-r} \quad \therefore\ C_\theta = C\sin^2\theta.$$

In the above case in the horizontal meridian

$$C_\theta = 4\sin^2 120° = 4\sin^2 60° = 4\left(\frac{\sqrt{3}}{2}\right)^2 = 3\text{ D}$$

In the vertical meridian $\theta' = 120° - 90°$.

$4\sin^2(120° - 90°) = 4\cos^2 120° = 4\cos^2 60° = 4(\tfrac{1}{2})^2 = 1$

In every case the power of a cylinder C set at axis θ has the power C $\sin^2\theta$ in the horizontal meridian, and C $\cos^2\theta$ in the vertical meridian.

In the above case the total power in the horizontal meridian is $3 + 3 = 6$ D, and in the vertical meridian $3 + 1 = 4$ D, so that the spherocylinder should be decentred $\frac{30}{6}$ or 5 mm. *in*, and $\frac{10}{4}$ or 2·5 mm. *up*.

Those who do not appreciate mathematical reasoning will be ready to accept the results, if they will try this simple experiment. Take a $+ 4$ D cylinder and test it with a lens measure; when the test is made in the plane axis the reading is 0, when at 30° from the plane axis the reading is $+ 1$, when at 60° the reading is $+ 3$.

The troublesome arithmetical work of squaring the sines and cosines of angles may be avoided by remembering that

$$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta) \text{ and } \cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta).$$

The values of all the sines and cosines are given on *p*. 235.

For instance, $\sin^2 20° = \tfrac{1}{2}(1 - \cos 40°) = \dfrac{1 - \cdot 766}{2} = \cdot 117$

$$\cos^2 15° = \tfrac{1}{2}(1 + \cos 30°) = \frac{1 + \cdot 866}{2} = \cdot 933$$

In no case need more than three decimal places be taken from the tables.

It may be urged that I have no right to treat this oblique ellipsoidal surface of the cylinder as a spherical surface for refraction. This would be a perfectly just criticism if the whole aperture of the lens were available, but the pupil of the eye only engages a very small area (about 2 mm. radius) near the geometrical centre of the lens. Also the radius of the cylinder is relatively very large, very rarely less than 200 mm., so that the small area engaged is practically indistinguishable from a spherical surface.

It is true that when during eccentric vision peripheral parts of spherocylinders are used, neither the astigmatism nor the heterophoria is accurately corrected, but this is always the case, and if it causes inconvenience the patient learns to avoid it by moving his head and not his eyes, so as always to use the central parts of his lenses.

Power of Two Oblique Cylinders.—If a cylinder of power C_1 set at an axis α be combined with a cylinder C_2 set at an axis β, the result of the combination will be a spherical lens of power D and a cylinder of power C′ set at an angle ϕ or

$$C_1\sin^2\alpha + C_2\sin^2\beta = D + C'\sin^2\phi$$

$$\text{or } C_1\tfrac{1}{2}(1 - \cos 2\alpha) + C_2\tfrac{1}{2}(1 - \cos 2\beta)$$
$$= D + C'\tfrac{1}{2}(1 - \cos 2\phi). \quad \ldots\ldots \quad (1)$$

$$D = \frac{C_1 + C_2 - C'}{2}$$

Now, however the combination of the two cylinders C_1 and C_2 be rotated, as long as the difference $\beta - \alpha$ remains constant the power of the combination will remain the same, i.e., D and C′ will be constant, it is only the values of the angles α, β and ϕ that will vary.

We must find C′ and ϕ. On subtracting (1) from $\frac{C_1}{2}+\frac{C_2}{2}=D+\frac{C'}{2}$ and multiplying by 2 we obtain

$$C_1 \cos 2\alpha + C_2 \cos 2\beta = C' \cos 2\phi.$$

Now what has been said above is equivalent to the statement that any arbitrary angle θ may be taken as the zero angle from which α and β are measured. We have then

$$C_1 \cos 2(\alpha-\theta) + C_2 \cos 2(\beta-\theta) = C' \cos 2(\phi-\theta).$$

On expanding these expressions and rearranging them according to their possession of the term cos 2ϕ or sin 2ϕ we obtain

$$(C_1 \cos 2\alpha + C_2 \cos 2\beta - C' \cos 2\phi) \cos 2\theta$$
$$+ (C_1 \sin 2\alpha + C_2 \sin 2\beta - C' \sin 2\phi) \sin 2\theta = 0.$$

Now as θ is entirely arbitrary, it is clear that each expression in brackets must be equal to 0.

$$\therefore\ C_1 \sin 2\alpha + C_2 \sin 2\beta = C' \sin 2\phi,$$
$$\text{and } C_1 \cos 2\alpha + C_2 \cos 2\beta = C' \cos 2\phi,$$

$$\therefore\ \tan 2\phi = \frac{C_1 \sin 2\alpha + C_2 \sin 2\beta}{C_1 \cos 2\alpha + C_2 \cos 2\beta} \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$\text{and } C' = \frac{C_1 \sin 2\alpha + C_2 \sin 2\beta}{\sin 2\phi} \text{ or } \frac{C_1 \cos 2\alpha + C_2 \cos 2\beta}{\cos 2\phi} \quad \ldots \quad (3)$$

whichever is the more convenient. If the resultant effect of two given cylinders is required, it is always best to find tan 2ϕ first.

EXAMPLE.—Find roughly the effect of the combination of a + 2 D cyl. ax. 15° and a − 1·5 D cyl. ax. 10°.

$$2 \sin 30° - 1{\cdot}5 \sin 20° = 1 - 1{\cdot}5\ ({\cdot}342) = {\cdot}487.$$
$$2 \cos 30° - 1{\cdot}5 \cos 20° = 1{\cdot}732 - 1{\cdot}5\ ({\cdot}94) = {\cdot}322.$$

$$\tan 2\phi = \frac{{\cdot}487}{{\cdot}322} = 1{\cdot}512 \risingdotseq \tan(n\pi + 57°).$$

$\therefore$ 2ϕ may be about 57° or 237°.

The expression for $C' = \dfrac{C_1 \sin 2\alpha + C_2 \sin 2\beta}{\sin 2\phi} = \dfrac{{\cdot}487}{\sin 2\phi}$

shows that if sin 2ϕ is negative, C′ will be negative, but if sin 2ϕ is positive, C′ will be positive.

$$\text{If } \sin 2\phi \text{ be } \sin 57^\circ,\ C' = \frac{\cdot 487}{\cdot 839} = \cdot 58$$

$$\text{if } \sin 2\phi \text{ be } \sin 237^\circ \text{ or } - \sin 57^\circ, \quad C' = - \cdot 58$$

$$D = \tfrac{1}{2}(2 - 1{\cdot}5 \pm \cdot 58) = + \cdot 54 \text{ or } - \cdot 04$$

i.e. $+ \cdot 54$ D sph. $- \cdot 58$ D cyl. ax. $118\tfrac{1}{2}^\circ$.
or $- \cdot 04$ D sph. $+ \cdot 58$ D cyl. ax. $28\tfrac{1}{2}^\circ$,

This special case has been taken in order to illustrate the phenomenon that was described on *p.* 51 when dealing with retinoscopy. It was there pointed out that when a cylinder of $-1{\cdot}5$ D was placed with axis 10° before an eye that really required $-$ 2 D cyl. ax. 15° the direction in which the light seemed to move across the pupil would be about in the 120° meridian, and might suggest that the axis should be moved to the 30° meridian so that the movement of the light should be exactly at right angles to the axis of the cylinder, i.e., that its axis of 10° was 20° wrong. It was, however, pointed out that under these circumstances the error in the position of the axis was magnified about four times, the real error being about one quarter of the apparent error, or 5°. If several examples are noted when correcting an astigmatism by retinoscopy and obtaining the axis in this easy way, the results of the subjective test with the final correction will convince the reader of its great value in finding the exact axis of the astigmatism. The little table on *p.* 53 is only roughly true, but it will be a help in practice.

Periscopic Lenses.—The object of these lenses is to enable the wearer to see distinctly when he is looking through an eccentric part of his spectacles.

As distinct vision is only possible when the image falls on the macula, and in examining the details of any object, say a picture, the eyes unconsciously glance from one point to another to fulfil this condition, it is only

necessary to make the macular image as distinct as possible. I have, therefore, paid no attention to the images formed on the peripheral parts of the retina; my aim has been to make the macular confusion circle less than that of a macular cone—the radius of which is ·001 mm.—when the eye is turned 30° from the primary position to view an object through an eccentric part of the correcting lens. It is a troublesome but not a very difficult matter to design a periscopic lens, bounded by spherical surfaces, which shall get rid of all the astigmatism induced by eccentric refraction, for all concave lenses used and for convex lenses up to + 7 D or so; but unfortunately the effective power of the lens is then so altered in many cases as to render them useless for eccentric vision at 30°.

Reading Lens.—It will be found that a + 4 D periscopic lens, which is quite satisfactory for distance, is not periscopic when used for reading, so that an entirely new table has to be made for periscopic lenses for close work. This is an important point that appears to have been neglected by the Jena School.

Before dealing with the design of periscopic lenses we must first see clearly what are the defects of the ordinary lens on eccentric vision that we wish to avoid. It will be well to take an extreme instance to show the defects in macular vision that arise from looking through an eccentric part of the ordinary biconvex lens. Suppose that an aphakic is given + 14 D for reading at a distance of 30 cm. from the plane of his glasses. If he rotates his eyes 30°, there will be formed by this peripheral part of his reading lens two focal lines, the first (v_1) at a distance of 39·02 mm. and the second (v_2) at a distance of 65·65 mm. on the ocular side of his lens.

Now $D = \dfrac{v - p}{vp}$ in metres.

Here p is the distance of the object, but it is not now

·3 m. for it is the slant distance at 30°, say ·3 sec 30° or ·34641 m. (This is not exact, for we are neglecting the prismatic effect of viewing an object through an eccentric part of the lens, but it will serve our purpose, although in this case p would be a trifle greater as the lens is convex.)

Then for the first focal line ($v_1 = -\cdot 03902$ m.).

$$D_1 = \frac{-\cdot 03902 - \cdot 34641}{-(\cdot 03902)(\cdot 34641)} = \frac{-\cdot 38543}{-\cdot 0135169} = +28\cdot 515 \text{ D.}$$

Similarly in the other meridian for the second focal line (v_2),

$$D_2 = \frac{-\cdot 06565 - \cdot 34641}{-(\cdot 06565)(\cdot 34641)} = \frac{-\cdot 41206}{-\cdot 0227418} = +18\cdot 119 \text{ D.}$$

The astigmatism of this biconvex lens when viewed eccentrically at an angle of 30° is $D_1 - D_2$, i.e., $+10\cdot 395$ D.

Further, we must find what should be the effective power of this lens with eccentric vision of 30°. As + 14 D is the correction for centric vision at a distance of ·3 m., the image must be formed at q, where

$$q = \frac{p}{1 - pD} = \frac{\cdot 3}{1 - 4\cdot 2} = -\cdot 09375 \text{ m.}$$

Now if p for eccentric vision be regarded as ·34641 m., since the image must be formed in the same place,

$$D = \frac{q - p}{qp} = \frac{-\cdot 09375 - \cdot 34641}{-(\cdot 09375)(\cdot 34641)} = \frac{-\cdot 44016}{-\cdot 0324476}$$

$$= +13\cdot 5534 \text{ D.}$$

Indeed, if we neglect the prismatic effect of the lens, for eccentric vision at this angle of 30°, its converging power should be made less by $-\cdot 4466$ D, for that is the lens that will form an image at ·3 m. of an object at ·34641 m.

Clearly, then, the biconvex lens when viewed eccentrically has not only a huge astigmatic error, but its power is far too great, so that the unfortunate patient would

be obliged to move his head for almost each word that he reads. On referring to the table, *p.* 207, it will be seen that by using the meniscus enjoined with its ocular surface of a curvature of — 4·5 D, the power of the lens is less than ·5 D too great and the astigmatism is less than 1·5 D. This is my very worst example in the Table of Convex Periscopic Lenses for reading, but as pointed out on *p.* 74, when reading an ordinary book only a range of movement of about 8° to either side of the middle line is required, so that for this purpose, where a solid angle of 16° instead of 60° is needed, every requisite will be satisfied. The tables have been calculated with the ocular surface given (which can be made with the tools in ordinary use), and the appropriate curvature for the anterior surface to give the assigned *back focal distance* required, according to the formula on *p.* 177; in the tables only three decimal places are given for the dioptric power of the anterior surface, as that will be enough to indicate the curvature desired. An error of ·125 D will make little difference except in the higher powers, but the ocular curvature should be as stated, so that really good periscopic lenses for the correction of simple myopia or of any hypermetropia up to + 7 D can be made with the ordinary tools at very slight expense. On *p.* 73 I have dealt with the great difficulties that arise when spherocylinders are required in periscopic form. It is a common practice to order the ocular surface to be toric on a — 6 D base, but it will be seen on looking at the table on *p.* 207 this cannot be satisfactory for convex lenses; possibly in one meridian the ocular curvature may be correct, but in all other meridians it will be hopelessly wrong. In fact, it is usually impossible to make perfect periscopic spherocylinders without a very large assortment of toric tools.

In the tables for convex periscopic lenses the column *t* gives the axial thickness of the lens of the assigned power

which is 40 mm. in diameter, r the radius of the retinal confusion-circle. As. gives the residual astigmatism with eccentric vision of 30°, while D′ gives this eccentric power of the periscopic lens. In the convex reading glasses D′ should be ·45 D less than the centric power (D) as explained on the previous page.

Concave spherical lenses are much more amenable to periscopic treatment as up to — 20 D or more in this form they give a trifling amount of residual astigmatism, and a tiny retinal confusion-circle smaller than a macular cone in every case. The axial thickness is always 1 mm., so that no calculation is required for determining the curvature of the anterior surface. I have, therefore, neglected to give the two columns for t and r. But it will be noted that for each power different curvatures must be given both to the anterior and ocular surfaces of the lens; consequently a concave periscopic sphero-cylinder should require special toric forms to be given to each surface. However, it should be remembered that when looking downwards ranging movements and convergence of the eyes should be discouraged in high myopia, so some caution should be exercised before prescribing periscopic lenses for high myopes with choroidal changes.

The mathematical investigation will now be given by which these tables were drawn up; it was first published in the July number of *The Ophthalmoscope*, 1914, and is an improvement on the method given in the former edition of this book. My most important results were published in the *Brit. Journ. of Ophthalmol.*, July, 1926; more complete tables are given on *p.* 207 with slight alterations in the values of r (the radius of the *retinal* circle of confusion), as I here use Gullstrand's constants for the eye instead of Tscherning's.

There are so many points to consider that it is often necessary to work out these tedious calculations several

times for one lens in order to find that which will best satisfy all the requisites that have been mentioned.

Determination of r.—Let *a* and *b* (*Fig.* 23) represent half the height and width of an eccentric pencil emerging from the lens at P and reaching two focal lines at F_1 and F_2. Let the position of the circle of least confusion be at D, and let its radius be *k*, the square of which is represented in the figure. Further, let H′ represent the first principal plane of the eye, and let *R* denote the radius of the equivalent pupil in this plane. Then it is required to construct the lens in such a fashion that the *retinal* image of this circle of confusion at D shall not be larger than the sectional area of a macular cone, the radius of which is ·001 mm.

We will first find an expression for *R*, the radius of the equivalent pupil in the first principal plane of the eye at H′ (*Fig.* 15). Imagine a line drawn from F″ touching the pupillary margin of the iris and cutting the principal plane at J, then H′J is *R*; if *y* be the radius of the pupil at A_1.

$$\frac{R}{y} = \frac{F''H'}{F''A_1} = \frac{F''H'' + H''H'}{F''A_0 + A_0A_1} = \frac{-22{\cdot}785 - {\cdot}254}{-24{\cdot}387 + 3{\cdot}6} = \frac{23{\cdot}039}{20{\cdot}787} \backsimeq 1{\cdot}1$$

The size of the pupil varies. I have assumed that its average radius (*y*) is 1·6 mm., so the radius (*R*) of its equivalent aperture at H′ would be 1·76 mm.

Now from the diagram (*Fig.* 23) it is seen that

$$(1)\quad \frac{DF_1}{k} = \frac{F_1H'}{R} \text{ or } \frac{DP - F_1P}{k} = \frac{F_1P - HP}{R}$$

$$(2)\quad \frac{F_2D}{k} = \frac{F_2H}{R} \text{ or } \frac{F_2P - DP}{k} = \frac{F_2P - HP}{R}$$

All these geometrical symbols (F_1P, F_2P, etc.) are taken in one direction from right to left, i.e., in the negative direction opposed to that of the incident light. Now both *k* and *R* merely denote the lengths of radii without any sense of direction, but as the angles at F_1 and F_2 are both

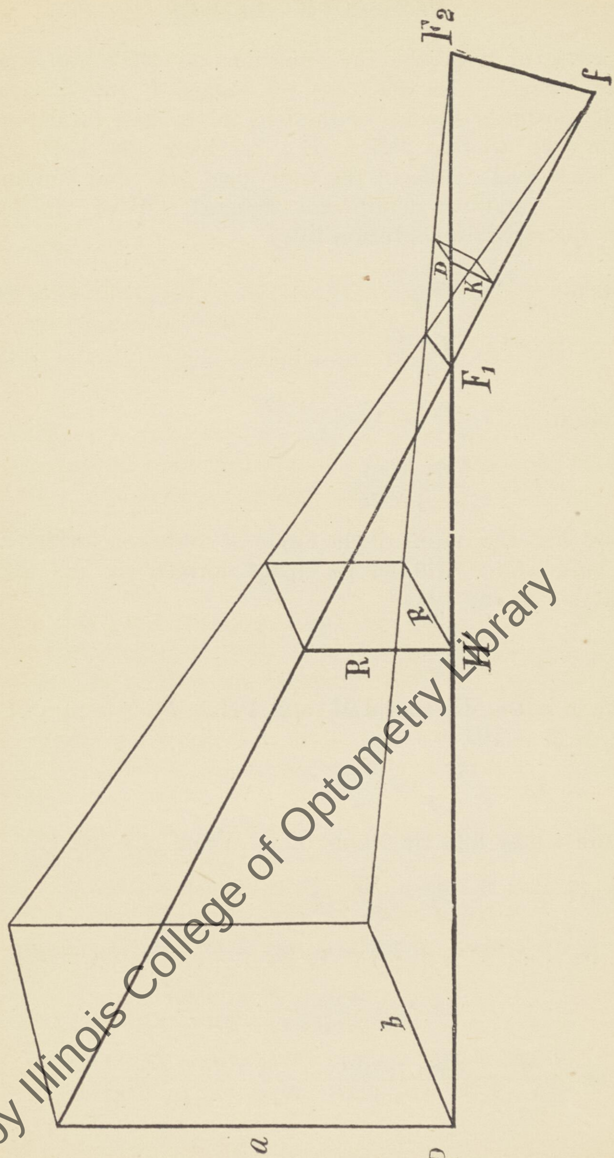

Fig. 23

clockwise or negative, the equations as they stand are quite correct. In this case we regard P the situation of the posterior surface of the lens in the first focal plane of the eye, so that PH$' = F'$; we have also $F_1P = v_1$, the back focal value of the first focal line, and similarly $F_2P = v_2$, and we denote the distance DP of the lens from the circle of confusion by x.

Then $$\frac{x - v_1}{k} = \frac{v_1 + F'}{R} \quad \text{.........................(1')}$$

$$\frac{v_2 - x}{k} = \frac{v_2 + F'}{R} \quad \text{.........................(2')}$$

By addition $$\frac{v_2 - v_1}{k} = \frac{v_1 + v_2 + 2F'}{R}$$

$$\therefore\ k = \frac{R\,(v_2 - v_1)}{v_1 + v_2 + 2F'}$$

Now k is the radius of the circle of confusion formed by the lens, and r will be its image formed by the axial refraction of the eye,

$$\therefore\ \frac{r}{k} = \frac{F'}{F' - p}$$

where p is the distance DH, and PH $= F'$, so $F' - p =$ PD $= -\,x$.

$$\therefore\ r = \frac{kF'}{F' - p} = \frac{-\,kF'}{x}$$

We must now find the value of x. From (1') and (2')

we have $$\frac{k}{R} = \frac{x - v_1}{v_1 + F'} = \frac{v_2 - x}{v_2 + F'}$$

$$\therefore\ x\,(v_2 + F' + v_1 + F') = v_1\,(v_2 + F') + v_2\,(v_1 + F')$$

$$\text{or } x = \frac{2v_1v_2 + F'\,(v_1 + v_2)}{v_1 + v_2 + 2F'}$$

$$\therefore\ r = \frac{-\,kF'}{x} = \frac{R\,(v_1 - v_2)\,F'}{v_1 + v_2 + 2F'}\ \frac{v_1 + v_2 + 2F'}{2v_1v_2 + F'\,(v_1 + v_2)}$$

$$\text{or } r = \frac{RF'(v_1 - v_2)}{2v_1v_2 + F'(v_1 + v_2)} = \frac{(1\cdot76)(17\cdot054)(v_1 - v_2)}{2v_1v_2 + 17\cdot054\ (v_1 + v_2)}$$

This is the formula used to give the values of r in the tables.

It may be noted that if the distance (HP) of the lens or the mirror from the eccentric aperture in the diaphragm be some value s instead of $-F'$ the radius of the circle of confusion is in every case $k = \dfrac{R(v_2 - v_1)}{v_1 + v_2 - 2s}$ and the distance (DP) of the lens from the circle of confusion is

$$x = \frac{2v_1v_2 - s(v_1 + v_2)}{v_1 + v_2 - 2s}$$

We have now to find the values of v_1 and v_2, which of course depend upon those of ϕ and ϕ' at the first surface and of ψ' and ψ at the second surface of the lens.

Determination of ϕ' and ψ and EF.—As the eye is assumed to rotate round the centre of motility M, the axial ray of the thin pencil from the object viewed that enters the pupil must pass through M. In *Fig.* 24 the eye is supposed to be rotated upwards through an angle θ in order to receive an incident pencil of parallel rays coming in the direction SE. On meeting the first surface EA_1 of the periscopic lens it is refracted in the direction EFQ; on meeting the second surface at F it is refracted as FM.

It is required to find the values of ψ, ϕ, and EF in terms of A_2M or k, C_2F or r_2, C_1E or r_1, A_1A_2 or t and θ to which given values can be assigned. The value of A_2M or k is taken to be 27 mm., and $\theta = 30°$, while t is 1 mm. in concave lenses and has a greater value in convex lenses.

From the diagram it is seen that

$$\frac{\sin C_2FM}{\sin FMC_2} = \frac{\sin C_2FM}{\sin A_2MF} = \frac{MC_2}{C_2F} = \frac{A_2C_2 - A_2M}{C_2F}$$

(1) $\therefore$ $C_2F \sin C_2FM = (A_2C_2 - A_2M) \sin A_2MF.$

(a) $\dfrac{\sin C_1EQ}{\sin EQC_1} = \dfrac{QC_1}{C_1E}$ $\quad\therefore\ QC_1 \sin A_2QF = C_1E \sin C_1EQ$

(b) and $\dfrac{\sin C_2FQ}{\sin FQC_2} = \dfrac{QC_2}{C_2F}$ $\quad\therefore\ QC_2 \sin A_2QF = C_2F \sin C_2FQ$

On subtracting (b) from (a) we obtain

$(QC_1 - QC_2) \sin A_2QF$ or

(2) $(A_1C_1 - A_1A_2 - A_2C_2) \sin A_2QF$
$= C_1E \sin C_1EQ - C_2F \sin C_2FQ.$

RE or $FE \sin RFM$, i.e., $FE \sin A_2QF = LE - L'F_1$

(3) $\therefore$ $FE \sin A_2QF = C_1E \sin A_1C_1E - C_2F \sin A_2C_2F.$

In order to obtain the *general* formulæ which shall be universally applicable to all periscopic lenses whether convex or concave, it is necessary to use symbols that carry inherently a sense of direction. As explained on *p.* 174 all the geometrical expressions must be taken in the same direction; on looking at the diagram we see that all the angles are measured clockwise, that is they are negative, so we will change the geometrical expressions so that they are all directed to the left.

Thus (1) becomes $C_2F \sin C_2FM$
$= (C_2A_2 - MA_2) \sin A_2MF$

(2) $C_1E \sin C_1EQ = (C_1A_1 - A_2A_1 - C_2A_2) \sin A_2QF$
$+ C_2F \sin C_2FQ$

(3) $FE \sin A_2QF = C_1E \sin A_1C_1E - C_2F \sin A_2C_2F$

The symbols that carry inherently a sense of direction are the following :—

$C_1ES_1 = \phi$	$A_2MF = \theta$	C_1E or $C_1A_1 = r_1$
$C_1EQ = \phi'$	$A_2C_2F = \theta - \psi$	C_2F or $C_2A_2 = r_2$
$C_2FQ = \psi'$	$A_2QF = \theta - \psi + \psi'$	$A_2M = k$
$C_2FM = \psi$	$A_1C_1E = \theta - \psi + \psi' - \phi'$	$A_1A_2 = t$

So (1) $r_2 \sin\psi = (r_2 + k) \sin\theta$

(2) $r_1 \sin\phi' = (r_1 + t - r_2) \sin(\theta - \psi + \psi') + r_2 \sin\psi'$

(3) $\mathrm{EF} \sin(\theta - \psi + \psi) = r_2(\theta - \psi) - r_1(\theta - \psi + \psi' - \phi')$

(3′) $\mathrm{EF} \cos(\theta - \psi + \psi') =$

$$t + r_1\{1 - \cos(\theta - \psi + \psi' - \phi') - r_2(1 - \cos\overline{\theta - \psi})\}.$$

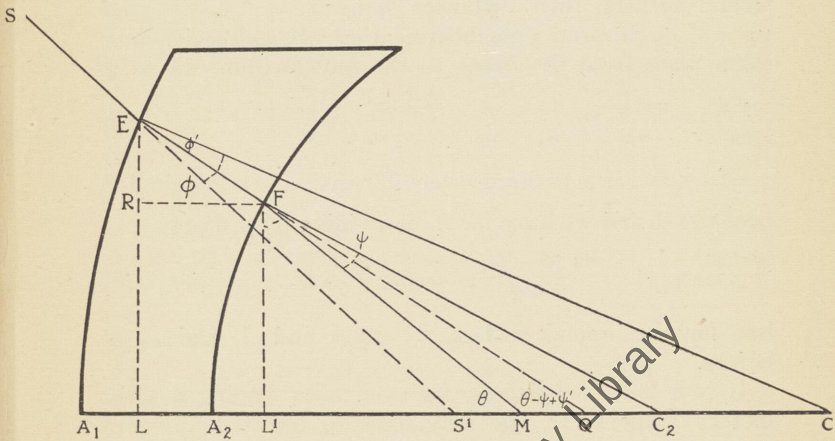

Fig. 24.

For plano-concave lenses it is easily seen that $A_2QF = \phi'$, and $A_1C_1E = 0°$, so by (3′) we get the modification below:—

For plano-concave lenses

(1) $r \sin\psi = (r + k) \sin\theta$

(2) $\phi' = \theta - \psi + \psi'$

(3) $\mathrm{EF} \cos\phi' = t - r(1 - \cos\overline{\theta - \psi})$

Determination of v_1 and v_2.—Having found the value of ψ from (1), the value of ψ' is determined by taking $\mu = 1{\cdot}523$, and hence ϕ' and ϕ are given by (2). The distances v' and v'' of the two focal lines due to the refraction at the first

surface of a point distant u (300 mm.) are given by the well-known formula :—

$$\frac{\mu \cos^2 \phi'}{v'} - \frac{\cos^2 \phi}{u} = \frac{\mu}{v''} - \frac{1}{u} = \frac{\sin (\phi - \phi')}{r_1 \sin \phi'} \quad \ldots\ldots\ldots\ldots (A)$$

Now for refraction at the second surface, we regard the focal lines just found as the objects for refraction at the second surface, their distances being $u_1 = v' + \text{EF}$, and $u_2 = v'' + \text{EF}$, and remembering that the refraction now takes place from the dense to the rare medium we write

$$\frac{\cos^2 \psi}{\mu v_1} - \frac{\cos^2 \psi'}{u_1} = \frac{1}{\mu v_2} - \frac{1}{u_2} = \frac{\sin (\psi' - \psi)}{r_2 \sin \psi} \quad \ldots\ldots\ldots\ldots (B)$$

Of course, if a periscopic lens for distance is required, $\frac{1}{u}$ becomes 0, and we have for A on dividing by μ simply

$$\frac{\cos^2 \phi'}{v'} = \frac{1}{v''} = \frac{\sin (\phi - \phi')}{r \sin \phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A')$$

but for the second surface we must find u_1 and u_2 as before.

It is a tedious business, for even after finding a satisfactory value for r, one may have to try with different curvatures to obtain a fairly satisfactory power on eccentric vision (*see p.* 197 on the biconvex lens).

In order to show these lenses in a comparable manner the value of the equivalent entrance-pupil (R, *Fig.* 23) has been taken as 1·76 mm. in all cases, and the value of r has always been calculated in the same way. As the distance of the second nodal point from the retina in a corrected aphakic eye is almost one-fourth as long again as that in a complete eye, about one-fourth must be added to the value of r to give its approximately correct value in an aphakic eye. Thus, in the table for distance lenses the value given for r with a + 10 D periscopic lens is given as ·00769 nearly; if given to an aphakic, the value of r would be about ·0096 mm.

CONVEX PERISCOPIC LENSES FOR DISTANCE.

SOLID ANGLE OF 60° ; $\mu = 1{\cdot}523$.

D	t	Ant. Surf.	Ocular Surf.	As.	r	D′
+ 1	1·38	+ 7·449 D	− 6·5 D	·0013 D	·000013	+ ·95 D
+ 2	1·77	+ 8·417 D	− 6·5 D	·0021 D	·000033	+ 1·89 D
+ 3	2·20	+ 9·371 D	− 6·5 D	·0027 D	·000042	+ 2·83 D
+ 4	2·54	+ 10·560 D	− 6·75 D	·0064 D	·000104	+ 3·76 D
+ 5	2·93	+ 11·490 D	− 6·75 D	·0042 D	·000082	+ 4·68 D
+ 6	3·33	+ 12·641 D	− 7·00 D	·0151 D	·000250	+ 5·59 D
+ 7	3·73	+ 13·536 D	− 7·00 D	·0681 D	·001136	+ 6·60 D
+ 8	4·14	+ 14·413 D	− 7·00 D	·149 D	·00258	+ 7·56 D
+ 9	4·55	+ 15·270 D	− 7·00 D	·259 D	·00469	+ 8·58 D
+ 10	4·98	+ 16·105 D	− 7·00 D	·428 D	·00769	+ 9·65 D
+ 11	5·41	+ 16·697 D	− 6·75 D	·641 D	·01178	+ 10·74 D
+ 12	5·86	+ 17·488 D	− 6·75 D	·908 D	·0171	+ 11·90 D
+ 13	6·32	+ 17·826 D	− 6·25 D	1·228 D	·0237	+ 13·08 D
+ 14	6·79	+ 18·152 D	− 5·75 D	1·579 D	·0313	+ 14·25 D

CONVEX PERISCOPIC LENSES FOR CLOSE WORK.

DISTANCE 30 CM. FROM SPECTACLE PLANE.

D	t	Ant. Surf.	Ocular Surf.	As.	r	D′
+ 1	1·38	+ 4·977 D	− 4·00 D	·0089	·00011	+ ·97 D
+ 2	1·77	+ 6·206 D	− 4·25 D	·0071	·00002	+ 1·92 D
+ 3	2·20	+ 7·175 D	− 4·25 D	·0013	·00002	+ 2·87 D
+ 4	2·54	+ 8·138 D	− 4·25 D	·0032	·00005	+ 3·80 D
+ 5	2·93	+ 9·088 D	− 4·25 D	·00004	·00006	+ 4·73 D
+ 6	3·33	+ 10·264 D	− 4·50 D	·00499	·00008	+ 5·63 D
+ 7	3·73	+ 11·421 D	− 4·75 D	·00994	·00015	+ 6·55 D
+ 8	4·14	+ 13·022 D	− 5·50 D	·01627	·00027	+ 7·43 D
+ 9	4·55	+ 13·668 D	− 5·25 D	·1404	·00181	+ 8·45 D
+ 10	4·98	+ 14·299 D	− 5·00 D	·2704	·00455	+ 9·49 D
+ 11	5·41	+ 15·140 D	− 5·00 D	·5015	·00866	+ 10·54 D
+ 12	5·86	+ 15·956 D	− 5·00 D	·7535	·01327	+ 11·64 D
+ 13	6·32	+ 16·532 D	− 4·75 D	1·0606	·01929	+ 12·90 D
+ 14	6·79	+ 17·090 D	− 4·50 D	1·4594	·02702	+ 13·98 D

CONCAVE PERISCOPIC LENSES.

Distance.			Distance 30 cm.	
— D	Ant. Surf.	Ocular Surf.	Ant. Surf.	Ocular Surf.
— 1	+ 6·25 D	— 7·25 D	+ 4·25 D	— 5·25 D
— 2	+ 5·75 D	— 7·75 D	+ 3·75 D	— 5·75 D
— 3	+ 5·00 D	— 8·00 D	+ 3·25 D	— 6·25 D
— 4	+ 4·75 D	— 8·75 D	+ 2·50 D	— 6·50 D
— 5	+ 4·25 D	— 9·25 D	+ 2·00 D	— 7·00 D
— 6	+ 3·75 D	— 9·75 D	+ 1·50 D	— 7·50 D
— 7	+ 3·25 D	— 10·25 D	+ 1·00 D	— 8·00 D
— 8	+ 2·75 D	— 11·75 D	+ ·50 D	— 8·50 D
— 9	+ 2·25 D	— 11·25 D	Plane	— 9·00 D
— 10	+ 2·00 D	— 12·00 D	— ·50 D	— 9·50 D
— 11	+ 1·50 D	— 12·50 D	— 1·00 D	— 10·00 D
— 12	+ 1·25 D	— 13·25 D	— 1·25 D	— 10·75 D
— 13	+ 1·00 D	— 14·00 D	— 1·75 D	— 11·25 D
— 14	+ ·75 D	— 14·75 D	— 2·00 D	— 12·00 D
— 15	+ ·50 D	— 15·50 D	— 2·00 D	— 13·00 D
— 16	+ ·25 D	— 16·25 D		
— 17	Plane	— 17·00 D		
— 20	Plane	— 20·00 D		

It will be noticed in the table for convex lenses the ocular concavity increases from — 6·5 D to — 7 D and then, after remaining stationary decreases to — 5·75 D. This is explained by the fact that the angle of incidence (φ) changes its sign at + 7 D, when it becomes positive, and for this reason an accurate periscopic lens cannot be made with spherical surfaces for powers of + 7 D or more for a solid angle of 60°.

The table is given in dioptric powers, as that is the way they are numbered in the workshops, but, of course, for calculation we must know the radius of curvature.

Since $\cdot 001\,\mathrm{D} = \dfrac{\mu - 1}{-r} = \dfrac{\cdot 523}{-r}$, $\qquad r = \dfrac{523}{-\mathrm{D}}$

Deviation of Prismospheres.—

For Distance.—In *Fig.* 25 ABE represents a prismosphere or a decentred lens, with its optical centre at O and its geometrical centre at D, opposite C, the centre of motility of the eye ; it is equivalent to the combination of the prism marked in dotted lines with a convex lens that is normally centred.

The deviation of the prismosphere at the point D, due to the decentration upwards DO of the optical centre, is the angle OFD or δ, while the deviation of the eye at C in order to receive the incident ("parallel") light from a distant source, must be DCL or ϕ.

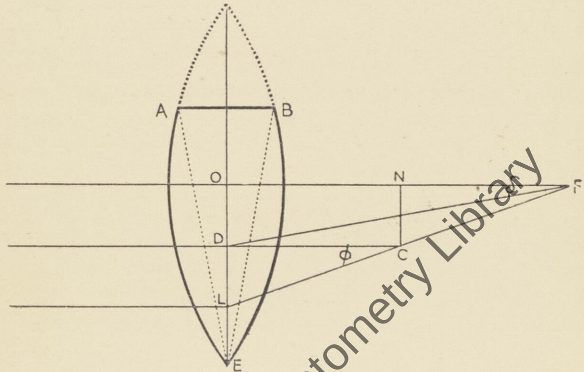

Fig. 25.

Now $\tan\,\delta = \dfrac{DO}{OF}$

and $\tan\,\phi = \tan\,NFC = \dfrac{CN}{NF} = \dfrac{DO}{OF - ON}$

$\therefore\ \dfrac{\tan\,\delta}{\tan\,\phi} = \dfrac{OF - ON}{OF} = 1 - \dfrac{ON}{OF}$

As all the lengths are measured in the positive direction (i.e., either in the direction of the incident light or upwards), and as the angles δ and ϕ are counterclockwise or

positive in the diagram, we can replace the geometrical expressions by their algebraic symbols and obtain a general formula that is universally true.

Here ON or DC $= k$, and OF $= f'$.

$$\frac{\tan \delta}{\tan \phi} = 1 - \frac{k}{f'} = 1 - k\mathrm{D} \text{ if } k \text{ is measured in metres.}$$

We see then that in *distant* vision

with convex prismospheres	$\phi > \delta$
when $\mathrm{D} = 0$, i.e., with a simple prism	$\phi = \delta$
with concave prismospheres	$\phi < \delta$

On *p.* 66 it was stated that an anisometrope using R + 3 D, L + 6 D, and viewing a distant object 15° below the horizontal plane would be obliged to depress his right eye 16° 15′ and his left eye 17° 44′ ; the following diagram (*Fig.* 26) will make this point clear.

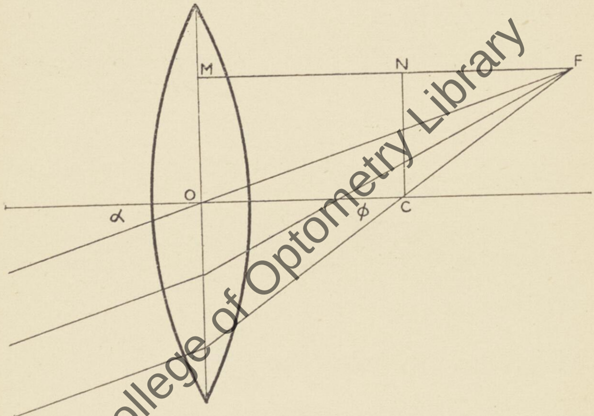

Fig. 26.

Light from a distant source at an angle α below the horizontal plane comes to a focus at F, so the eye must rotate downwards through the angle ϕ to receive it.

Now $\tan \alpha = \tan \text{MFO} = \dfrac{\text{OM}}{\text{MF}}$

and $\tan \phi = \tan \text{NFC} = \dfrac{\text{CN}}{\text{NF}} = \dfrac{\text{OM}}{\text{MF} - \text{MN}}$

$$\frac{\tan \alpha}{\tan \phi} = \frac{\text{MF} - \text{MN}}{\text{MF}} = 1 - \frac{k}{f'} \text{ or } 1 - k\text{D},$$

if we are allowed to regard MF as equal to f'; this is strictly only permissible if the angle is small.

Returning to the problem on *p.* 66, as $k = \cdot 027$ m., for the Right eye $1 - 3(\cdot 027) = \cdot 919$

and for the Left eye $1 - 6(\cdot 027) = \cdot 838$

Right $\tan \phi = \dfrac{\tan 15°}{\cdot 919} = \tan 16° \; 15' \; 18''$

Left $\tan \phi = \dfrac{\tan 15°}{\cdot 838} = \tan 17° \; 43' \; 54''$

and the difference is 1° 28′ 36″. It is dishonest to pretend to give the result in seconds when we are neglecting the astigmatism caused by the oblique refraction; but I give the result in detail in order to show the method. The real difference would be more than $1\frac{1}{2}°$.

For Close Work.—Let MOD in *Fig.* 27 represent the spectacle plane, O being its optical centre, and D its geometrical centre, opposite C, the centre of motility of the eye. Let P be the object viewed at the distance PD (or p) and let its image be formed at Q, and let QM be denoted by q.

As before, $\tan \delta = \tan \text{OFD} = \tan \text{MQD} = \dfrac{\text{DM}}{\text{MQ}}$

$$\tan \phi = \tan \text{NQC} = \frac{\text{CN}}{\text{NQ}} = \frac{\text{DM}}{\text{MQ} - \text{MN}}$$

$$\therefore \frac{\tan \delta}{\tan \phi} = \frac{\text{MQ} - \text{MN}}{\text{MQ}} = 1 - \frac{\text{MN}}{\text{MQ}} = 1 + \frac{k}{q}$$

$$= 1 + \frac{k\,(1 - p\text{D})}{p} = 1 + \frac{k}{p} - k\text{D}.$$

$$\therefore \tan \phi = \frac{\tan \delta}{1 + \dfrac{k}{p} - k\text{D}}$$

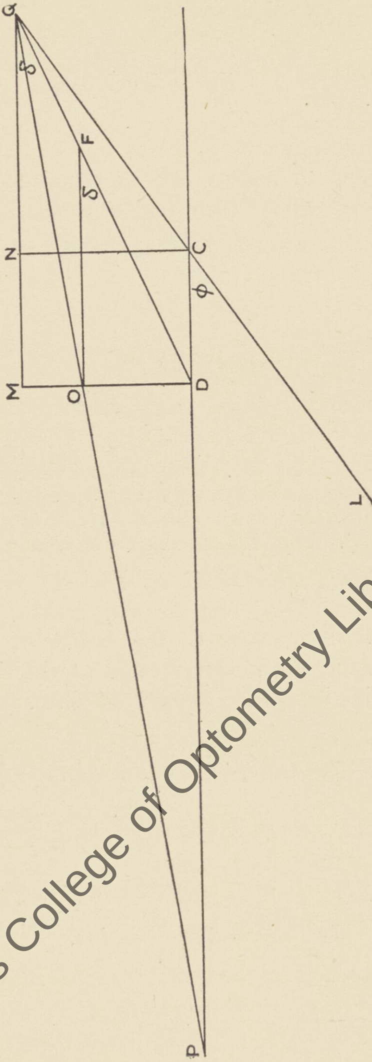

Fig. 27

From this formula it is seen that with concave lenses, when D is negative, ϕ is always less than δ. With convex lenses if $p = f'$, $\frac{k}{p} = k\text{D}$, so $\phi = \delta$.

if $p < f'$, $\phi < \delta$,

if $p > f'$, $\phi > \delta$.

For a simple prism without a lens, one puts $\text{D} = 0$ and one gets the form $\tan \phi = \dfrac{\tan \delta}{1 + \frac{k}{p}}$ which was practically given on *p*. 91.

When the lens is normally centred and the object is at an angle α, and on either side of the horizontal plane, or of the median vertical plane through O, as in reading, the formula still holds good, as may be seen by drawing a diagram similar to *Fig.* 27, only placing C at the intersection of NC and OF and joining QC. The angle MQO or DPO now represents α, and the angle NQC represents ϕ.

There is a small point that arises which may create some difficulty; if convergence is being considered it is clear that a rotation inwards of each eye occurs, so that there is a positive rotation of the right eye and a negative rotation of the left eye. Convergence is universally regarded as positive by ophthalmologists, so that it is well to regard the rotations of the right eye as the standard rotations.

As has already been pointed out, these results are only approximate, but are in the most convenient form for practical use. The table (*p*. 215) gives the value of the factors that are required for finding the real deviation of the eye when provided with prismospheres for viewing distant or close objects. In cases of heterophoria the tests are made with prisms, and that prism is given with the appropriate correction which gives relief, so reference to this table will rarely be necessary in such cases.

Anisometropia and Bifocals.—When considering this problem on *p.* 77 the reasoning was as follows: As the patient is to look through the geometric centres of the wafers, which are situated 5·75 mm. below the centres of the field lenses, each of the latter may be considered virtually decentred 5·75 mm. upwards for near work. Denote this decentration in millimetres by l.

We know that $l = \frac{10\,\mathrm{N}}{\mathrm{D}}$; in the case of the right lens D′ (of power + 3 D), l or 5·75 mm. $= \frac{10\,\mathrm{N}}{3}$.

$$10\,\mathrm{N} = 17{\cdot}25\ \triangledown \text{ or } \mathrm{N} = 1{\cdot}725\ \triangledown.$$

If the right wafer (+ 2 D) is not itself decentred, the eye, looking through its geometric centre, is looking through a + 5 D lens combined with a 1·725 ▽ prism, base up. What must be the decentration (l') of the left wafer in order to make the prism virtually carried by the left combination exactly N ▽, or 1·725 ▽ ?

Clearly $10\ \mathrm{N} = \mathrm{D}'l = \mathrm{D}l + \mathrm{d}l'$

$$\therefore\ l' = \frac{(\mathrm{D}' - \mathrm{D})l}{\mathrm{d}} = \frac{(3-6)5{\cdot}75}{2} = -\ 8{\cdot}625 \text{ mm. (down).}$$

The table shows the factors which multiplied into tan δ give tan ϕ (the deviation of the eye) approximately when provided with any lens.

But from the table it will be seen that as the power of the two combinations is different, the deviation of the two eyes will also be different.

The power of the combination for the right eye is + 5 D, and that for the left eye is + 8 D, so we must have

$$1{\cdot}047\ \mathrm{D}'l = 1{\cdot}144\ \mathrm{D}l + 1{\cdot}144\ \mathrm{d}l'$$

$$\therefore\ l' = \frac{1{\cdot}047\ \mathrm{D}' - 1{\cdot}144\ \mathrm{D}}{1{\cdot}144\ \mathrm{d}}\,l = \frac{\frac{1{\cdot}047}{1{\cdot}144}\,3 - 6}{2}\ (5{\cdot}75).$$

$$l' = -\ (1{\cdot}6272)\ (5{\cdot}75) = -\ 9{\cdot}356 \text{ mm. (downwards).}$$

The left wafer should then really be decentred ·731 mm. more than previously stated.

It is seen that if the combinations are positive in power the rough formula on *p.* 77 gives an under-correction, but if the power is negative it will give an over-correction. The table will be of great use in ordering bifocals for those whose anisometropia is greater than 2 D.

Deviation of Prismospheres for Near and for Distant Objects.

p	∞	30 cm.	p	∞	30 cm.
	$\frac{1}{1-kD}$	$\frac{1}{1+\frac{k}{p}-kD}$		$\frac{1}{1-kD}$	$\frac{1}{1+\frac{k}{p}-kD}$
D			D		
+ 1	1·028	·941	— 1	·974	·895
+ 2	1·057	·965	— 2	·949	·874
+ 3	1·088	·991	— 3	·925	·854
+ 4	1·121	1·018	— 4	·903	·835
+ 5	1·156	1·047	— 5	·881	·816
+ 6	1·193	1·078	— 6	·861	·799
+ 7	1·233	1·110	— 7	·841	·782
+ 8	1·276	1·144	— 8	·822	·766
+ 9	1·321	1·181	— 9	·804	·750
+ 10	1·370	1·220	— 10	·787	·735
+ 12	1·479	1·305	— 12	·756	·707
+ 14	1·608	1·404	— 14	·726	·681
+ 16	1·761	1·520	— 16	·698	·657
+ 18	1·946	1·656	— 18	·673	·635
+ 20	2·174	1·818	— 20	·649	·613

Objective Tests of Ametropia.—When the eye is examined with the ophthalmoscope by the indirect method it was stated on *p.* 43 that on removing the lens from the eye in myopia the size of the image of the fundus was increased, but that in hypermetropia it was diminished. The adjoining diagram will explain this peculiarity.

Myopia.—As the fundus in myopia lies behind the focus of the eye, it will, when illuminated, form a real inverted image (I) in front of the eye, without the lens (O), say at P′ its punctum remotum (*Fig.* 28).

On interposing a + 13 D convex lens (focus 76·92 mm.) near the eye at O, the emergent light will converge still more, so as to form an image at Q. This inverted image (i) is what is seen with the ophthalmoscope. In the diagram, the incident light reflected from the ophthalmoscope is passing from right to left, whereas the light from the fundus is travelling from left to right; we will regard this last as the positive direction.

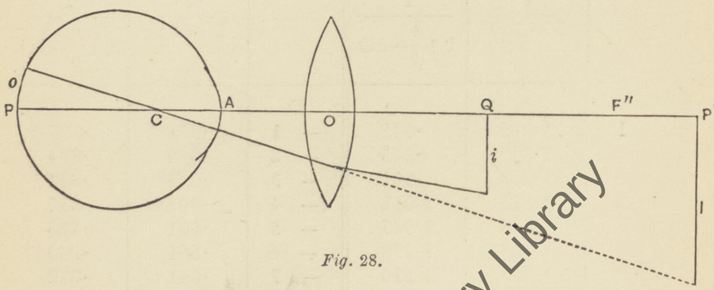

Fig. 28.

Let us imagine that − 9 D of myopia is present, then from *p.* 165 we know that the eye is l or 9 × ·39 mm., say 3·5 mm., longer than normal. With the simplified eye *p.* 157, CA = 5·73, and in this myopic eye

$$CP = -(F' + l) = -17{\cdot}05 - 3{\cdot}5 = -20{\cdot}55 \text{ mm.}$$

Suppose the − 9 D lens to be worn at L in the first focal plane of the eye, i.e. 17·05 mm. from A; as we know that the distance P′L of the lens from the far point P′ must be the second focal distance ϕ'' of the correcting lens.

$$P'A = P'L + LA = -\frac{1000}{-9} + 17{\cdot}05 = 111{\cdot}11 + 17{\cdot}05$$

But as in the diagram we are considering light travelling from the eye to the space in front, we must regard AP′ as +128·16, so CP′=CA+AP′=5·73+128·16=133·89.

Hence $\frac{I}{o} = \frac{\text{CP}'}{\text{CP}} = \frac{133{\cdot}89}{-\ 20{\cdot}55}$

Now I will be the object for the convex lens and i will be its image

and $\frac{i}{I} = \frac{f'}{f' - p} = \frac{f'}{f' - \text{P}'\text{O}} = \frac{f'}{f' + \text{AP}' - \text{AO}}$

and as f' of the convex lens is 76·92 mm.

$$\frac{i}{I} = \frac{76{\cdot}92}{76{\cdot}92 + 128{\cdot}16 - \text{AO}} = \frac{76{\cdot}92}{205{\cdot}08 - \text{AO}}$$

$$\therefore\ \frac{i}{o} = \frac{I}{o}\,\frac{i}{I} = \frac{133{\cdot}89}{-\ 20{\cdot}55}\ \frac{76{\cdot}92}{205{\cdot}08 - \text{AO}} \approx \frac{-\ 501{\cdot}16}{205{\cdot}08 - \text{AO}}$$

It is clear, then, that in myopia if the distance AO of the lens from the eye be increased, the value of i will be increased; the negative sign shows that the image is inverted. The table (*p.* 221) gives the magnification of the image in myopia, hypermetropia, and emmetropia when the lens is held at different distances from the eye.

Hypermetropia.—Suppose that the eye is hypermetropic to the extent of $+$ 6 D, then $l = -\ 6({\cdot}39) = -\ 2{\cdot}34$ mm. or the eye is too short by 2·34 mm.

so $\text{CP} = -\ (F' + l) = -\ 17{\cdot}05 + 2{\cdot}34 = -\ 14{\cdot}71.$

The image I of P will therefore now be formed at some point P″ behind the retina.

$$\text{AP}'' = \text{AL} - \text{P}''\text{L} = 17{\cdot}05 - \frac{1000}{6} = 17{\cdot}05 - 166{\cdot}66.$$

$$\text{CP}'' = \text{CA} + \text{AP}'' = 5{\cdot}73 - 149{\cdot}61 = -\ 143{\cdot}88.$$

$$\frac{I}{o} = \frac{\text{CP}''}{\text{CP}} = \frac{-\ 143{\cdot}88}{-\ \ 14{\cdot}71}$$

$$\frac{i}{I} = \frac{f'}{f' - p} = \frac{f'}{f' - \text{P}''\text{O}} = \frac{f'}{f' - \text{P}''\text{A} - \text{AO}}$$

i.e., $\frac{i}{I} = \frac{76{\cdot}92}{76{\cdot}92 - 149{\cdot}61 - \text{AO}} = \frac{76{\cdot}92}{-\ 72{\cdot}69 - \text{AO}}$

$$\frac{i}{o} = \frac{I}{o}\,\frac{i}{I} = \frac{143{\cdot}88}{14{\cdot}71}\ \frac{76{\cdot}92}{-\ 72{\cdot}69 - \text{AO}} \approx \frac{752{\cdot}36}{-\ 72{\cdot}69 - \text{AO}}$$

In hypermetropia, then, if the distance AO be increased, the magnification of the image will be diminished (see table, *p.* 221).

A glance at the diagram (*Fig.* 28) will explain the the point mentioned on *p.* 42. It was said that when the observer reflecting the light from the mirror at a distance sees an image of the fundus without a lens, either hypermetropia or myopia is present. If myopia, the vessels of the fundus appear to move in the opposite direction to the observer's head ; if hypermetropia, they appear to move in the same direction. The explanation usually given—that it depends on the inverted or the erect image formed in the two cases—is faulty. The illuminated pupil and its defined margin is taken to be fixed in position. In high myopia a real inverted image is formed between the observer and the patient's pupil ; consequently, on the observer's moving his head to the right the vessels are seen moving to the left of the pupillary area. But in hypermetropia, as the image is formed some way behind the patient's pupil, when the observer moves his head to the right the vessels are seen moving to the right of the illuminated pupillary area. Try holding up your finger and looking at a distant picture ; on moving your head to the right, if the picture be fixed by your eye, your finger will appear to move to the left (myopia) ; if your finger be fixed, the picture will appear to move to the right (hypermetropia).

Emmetropia.—In this case light from the point P (*Fig.* 28) after passing through the cornea will emerge as a beam parallel to the axis PCA until it encounters the lens at O, when it will converge to the focus F″. The cone of light that originates at E (*Fig.* 29) will emerge as a beam parallel to the secondary axis ECK and after leaving the lens will converge to a point (E) on the lower part of the plane through *f″*. As the light emerges in parallel beams the image will always be formed in the second focal plane

of the lens, and so its size will always be the same whatever the position of the lens, although of course the image will be dimmer the further the lens is removed from the eye.

From *Fig.* 29 it is seen that

$$\frac{i}{o} = \frac{OL}{AB} = \frac{F'O}{F'A} = \frac{76 \cdot 92}{-17 \cdot 05} = -4 \cdot 51.$$

In every case, whatever be the refraction of the eye, the weaker the lens, the greater will be the magnification, but the power given (+ 13 D) is almost invariably used. A power weaker than + 11 D is inconvenient, as the

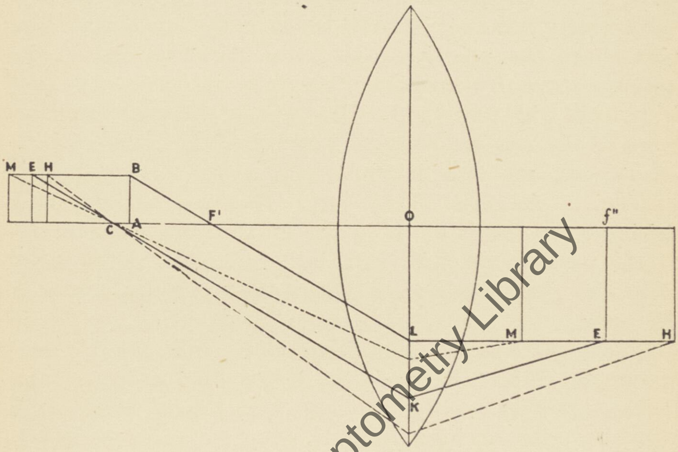

Fig. 29.

Note F′O = LE = 76.92 mm.
ECK is || BF′L

CA 5·73 mm.
AF′ 17·05 mm.

observer must keep his eye at too great a distance from the lens. In cases of aphakia it will be found that the image is formed at such a distance behind the lens that it may be difficult or impossible to see it. A stronger lens should then be used, say + 20 D, or by putting up a strong convex lens behind the ophthalmoscope a satisfactory examination can generally be made.

There is one position of the lens in which the magnification of the image is always the same whatever be the refraction of the eye. This occurs when the lens is held at its focal distance from the anterior focal plane of the eye.

In *Fig.* 29 AB represents the Unit Plane, F′ the anterior focus of the eye, and O the centre of the lens. If the lens be + 13 D, $F'O = Of'' = 76{\cdot}92$ mm. and $AF' = 17{\cdot}05$ mm. so $AO = 93{\cdot}97$ mm. or about 3·7 in.

Let M, E, and H represent corresponding points at the back of a myopic, emmetropic, and hypermetropic eye respectively ; the ray MEHB which in the eye is travelling parallel to the principal axis must pass through F′ the anterior focus of the eye, and continue in this course until it reaches the lens at L. But F′ is the first principal focus of the lens, for its focal length is F′O ; consequently on leaving the lens it must pursue the course LMEH parallel to the axis of the lens. The cone of light BEC arising from the point E in the emmetropic eye emerges as the parallel beam BL, CK and converges to the point E in the plane through *f″* the second principal focus of the lens.

In myopia the cone of light BMC will emerge as a converging beam (BL and the dotted line) until it meets the lens, when it will be brought to a nearer focus at M. But the cone of light from H will emerge as a slightly diverging beam, and so after refraction through the lens will converge to the more distant point H.

In all cases it is seen that the images are of exactly the same size although they are formed at different distances from the lens.

Below is given a little table of the magnification obtained when using the indirect method of ophthalmoscopy with a lens of + 13 D in the cases of a myope of − 9 D, an emmetrope, and a hypermetrope of + 6 D, when the lens is held in the first focal plane of the eye,

at the focal distance of the lens from A, and finally at the sum of these two distances. In the last case the magnification is, as we have seen, exactly the same, being − 4·51, and this may be taken as a rough indication of the magnification obtained when examining most eyes by the indirect method.

MAGNIFICATION BY INDIRECT METHOD WITH LENS AT DIFFERENT DISTANCES FROM THE EYE.

	F'	f'	$F' + f'$
AO	17·05	76·92	93·97
− 9 D	− 2·67	− 3·91	− 4·51
Em.	− 4·51	− 4·51	− 4·51
+ 6 D	− 8·38	− 5·03	− 4·51

Minimum Visual Angle.—This, as explained on *p.* 28, is a problem that is still shrouded in obscurity, as Light Perception introduces difficulties that are not amenable to optical treatment. However, if we regard the minimum visual angle as twice the angle subtended at the posterior nodal point by one macular cone (diameter ·002 mm.) it may be treated as follows.

Let *o* in *Fig.* 28 represent a macular cone, and as in emmetropia CP = 17·054 mm.

$$\tan \alpha = \frac{\cdot 002}{17\cdot 054} = \tan 24\cdot 14''.$$

$$2\ \alpha = 48\cdot 28''.$$

On this basis a value of $48\frac{1}{2}''$ may be taken as the minimum visual angle.

APPENDIX.

SERVICE REGULATIONS.

COMMISSIONS IN THE ARMY (1927).

"THE Army Test Types will be used for the test for distant vision, without glasses at a distance of 20 feet; and for the test for near vision without glasses, at any distance selected by the candidate."

Standard I.

	Right Eye.	*Left Eye.*
Distant Vision	$V = \frac{6}{6}$.	$V = \frac{6}{6}$.
Near vision	Reads 0·6	Reads 0·6.

Standard II.

	Better Eye.	*Worse Eye.*
Distant Vision	$V = \frac{6}{6}$.	V without glasses = not below $\frac{6}{60}$; and after correction with glasses = not below $\frac{6}{24}$.
Near vision	Reads 0·6.	Reads 1.

Standard III.

	Better Eye.	*Worse Eye.*
Distant Vision	V without glasses = not below $\frac{6}{60}$, and after correction with glasses = not below $\frac{6}{6}$.	V without glasses = not below $\frac{6}{60}$; and after correction with glasses = not below $\frac{6}{12}$.
Near Vision	Reads 0·8.	Reads 1.

Each eye must have a full field of vision as tested by hand movements.

Recruits for the Army.

There is no Near Vision Test for recruits, for the Distant Test the recruit must keep the eye that is being tested wide open. Either condition (*a*) or condition (*b*) must be satisfied.

	Right Eye.	*Left Eye.*
(*a*)	V = not less than $\frac{6}{18}$ without glasses.	V = not less than $\frac{6}{18}$ without glasses.
(*b*)	V = $\frac{6}{6}$ without glasses.	V = not less than $\frac{6}{36}$ without glasses.

The Royal Navy (1927).

In *all* cases colour sense must be normal. The Lantern test instead of Holmgren's wools has been officially recognized for the final examination.

	Near Vision.	*Distant Vision.*
Cadets (Dartmouth)	D. 0·6	$\frac{6}{6}$ each eye.
Special Entry—		
Cadets and Marines	D. 0·6	$\frac{6}{6}$, $\frac{6}{12}$.
Engineer Branch	D. 0·6	$\frac{6}{6}$, $\frac{6}{12}$.
Other Branches	D. 0·6	Not less than $\frac{6}{60}$ each eye; with correction $\frac{6}{6}$ each eye.

In every case the above standards must be obtained without the use of glasses, and vision must be capable of being corrected to $\frac{6}{6}$ with the aid of suitable glasses.

The Royal Air Force (1927).

(*a*) The standard of minimum acuteness of vision for distance is $\frac{6}{6}$ in each eye.

Where a candidate is otherwise especially fit, the eye specialist may recommend acceptance when the

acuteness of vision is $\frac{6}{9}$ in each eye provided that with glasses it is $\frac{6}{6}$ in each eye.

Hypermetropia of + 2D or more will disqualify.

(*b*) Both eyes must have good binocular fields of vision as tested by hand movements.

(*c*) There must be good binocular fusion and good balance of the ocular muscles.

(*d*) There must be normal colour vision according to the Board of Trade standards.

NON-MILITANT SERVICES.

Masters and Mates in the Mercantile Marine.

The Letter Test.

Special Test Types are used for these services, which are placed at a distance of 16 feet from the candidate; spectacles and other artificial aids to vision are not allowed.

The candidate will have the option of using either eye separately or both eyes together.

If he has read correctly nine of the twelve letters in the sixth line, and eight of the fifteen letters in the seventh line, he has passed the 'Letter Test.'

The Lantern Test.

The candidate must be kept in a darkened room for at least a quarter of an hour before he is required to undergo this test.

The lantern allows three lights to be visible, one large and two small lights; twelve glasses of three different colours—red, white, and green—are provided to be placed before these lights.

If a red light is mistaken for green, or a green light for red, the candidate has failed.

If he confuses red and white, or green and white, he must submit to further tests by the Principal Examiner before he can be certified as having passed the 'Lantern Test.'

The Departments of Forest, Survey, Telegraph, Factories, and for other Artificers.

1. Myopia in one or both eyes not exceeding 2·5 D does not disqualify, if with correcting glasses, not exceeding 2·5 D, the sight of one eye is $\frac{6}{6}$ and that of the other is $\frac{6}{9}$, there being normal range of accommodation with the glasses.

2. Similarly, myopic astigmatism does not disqualify, provided that the lens or the combined spherical and cylindrical lenses do not exceed 2·5 D, and the other conditions in (1) are fulfilled, there being no evidence of progressive disease in the choroid or retina.

3. Total hypermetropia not exceeding 4 D does not disqualify provided that (when under the influence of atropine) the sight of one eye is $\frac{6}{6}$ and that of the other is $\frac{6}{9}$ with + 4 D or any lower power.

4. Similarly, hypermetropic astigmatism does not disqualify, provided that the lens or combined lenses do not exceed 4 D, and that the sight of one eye is $\frac{6}{6}$, and that of the other is $\frac{6}{9}$, with or without such lens or lenses.

5. Corneal nebula does not disqualify, provided that the sight of one eye is not less than $\frac{6}{12}$, while the other eye is emmetropic. Defects of vision arising from pathological or other changes in the deeper structures of either eye, which are not referred to in the above rules, may disqualify.

6. Squint or any morbid condition, subject to the risk of aggravation or recurrence, in either eye, may disqualify. The existence of imperfection of colour sense will be noted on the candidate's papers.

Public Works Department, and Superior Establishments, Railway Department.

The Regulations for this branch of the Service are identical with those for the Departments of Forest, etc., preceding, except that a higher degree (3·5 D) of myopia and myopic astigmatism is allowed instead of 2·5 D,

and that there is this addition to regulation (6). Any imperfection of the colour sense is a disqualification for appointment to the Engineering Branch of the Railway Department, or as Assistant Superintendent in the Traffic Department.

The Indian Medical Service and the Police Department.

The Regulations for these two branches of the service are identical with those for Commissions in the Army.

The Indian Pilot Service, and Appointments as Guards, Engine-drivers, Signalmen, and Pointsmen on Railways.

A candidate is disqualified :—

1. Unless both eyes are emmetropic, his acuteness of vision and range of accommodation being perfect.
2. By any imperfection of his colour sense.
3. By strabismus or any defective action of the exterior muscles of the eyeball.

The Indian Marine Service, including Engineers and Firemen.

A candidate is disqualified :—

1. By an error of refraction in one or both eyes which is not neutralized by a concave or by a convex glass, 1 D lens or some lower power.
2. By any imperfection of his colour sense.
3. By strabismus or any defective action of the exterior muscles of the eyeball.

Post Office, Railway, Constabulary.

No fixed standard of vision has been officially laid down for entering any one of these services; the decision rests with the Medical Officer in charge of that department. In the interests of the community a minimum standard of vision should be specified, which every candidate for entrance into either the Railway or the Constabulary Service in any part of the country should pass.

OPTICAL FORMULÆ.

A. Universally True—

(*a*) For thin centric pencils—

$$(1)\ \frac{f'}{p} + \frac{f''}{q} = 1$$

$$\therefore\ p = \frac{qf'}{q - f''} \text{ and } q = \frac{pf''}{p - f'}$$

$$(2)\ \frac{i}{o} = \frac{f'}{f' - p} = \frac{f'' - q}{f''}$$

$$\therefore\ (f' - p)\ (f'' - q) = f'f''$$

(*b*) For oblique pencils when v_1 and v_2 carry the same sign

$$v_2 \gtreqless v_1 \quad \text{as} \quad u \gtreqless \mu v_1$$

(*c*) Circle of least confusion (*See pp.* 200–3)

$$k = \frac{R(v_2 - v_1)}{v_1 + v_2 - 2s}$$

$$x = \frac{2v_1v_2 - s(v_1 + v_2)}{v_1 + v_2 - 2s}$$

B. Refraction at a Spherical Surface—

When μ_0 is the index of the medium in which the source of light lies, and μ' is the index of the refracting medium—

$$\text{(i)}\ f' = \frac{-\mu_0 r}{\mu' - \mu_0}$$

$$\text{(ii)}\ f'' = \frac{\mu' r}{\mu' - \mu_0} = \frac{-\mu'}{\mu_0} f' \quad \therefore\ f'' + f' = r$$

ECCENTRIC PENCILS—

$$\frac{\mu'}{\mu_0} \frac{\cos^2 \phi'}{v_1} - \frac{\cos^2 \phi}{u} = \frac{\mu'}{\mu_0 v_2} - \frac{1}{u} = \frac{\sin(\phi - \phi')}{r \sin \phi'}$$

C. **Lenses**—

(i) Thin, $\frac{1}{f''} = \frac{\mu' - \mu_0}{\mu_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = -\frac{1}{f'} = -\text{D}.$

(ii) Thick, $\frac{1}{f''} = \frac{\mu' - \mu_0}{\mu_0 r_1 r_2}\left(r_2 - r_1 - \frac{\mu' - \mu_0}{\mu'}t\right)$

$$= -\frac{1}{f'} = -\text{D}$$

Cardinal Points—

$$\text{When } K = t + f_1'' - f_2',$$

$$h' = \frac{f_1't}{K},\ h'' = \frac{f_2''t}{K},\ F' = \frac{-f_1'f_2'}{K},\ F'' = \frac{f_1''f_2''}{K}$$

In all lenses $f_1'' > f_1'$, and $f'_2 > f_2''$ numerically.

In Bi-lenses f_1' and f_2' carry the same sign, while the sign carried by f_1'' and f_2'' is opposite to that of f_1' and f_2'.

In Menisci both f_1' and f_2'' carry the same sign, but the reversed sign is carried by f_1'' and f_2'.

In complex systems,

$$t = \text{H}_a''\text{H}_b',\ h' = \text{H}'\text{H}_a',\ h'' = \text{H}''\text{H}_b''.$$

Dioptres—

When f' given in centimetres,

$$\text{D} = \frac{100}{f'} \qquad \text{or } \cdot 01\ \text{D} = \frac{1}{f'}$$

when f' in millimetres, $\cdot 001\ \text{D} = \frac{1}{f'}$

Cardinal Points—

(1) *Two thin lenses* d_1 *and* d_2 *in air, interval t metres*:—

$$\text{D} = \text{d}_1 + \text{d}_2 - t\text{d}_1\text{d}_2, \text{ and } \frac{1}{K} = \frac{-\text{d}_1\text{d}_2}{\text{D}}$$

$$h' = -\frac{t\text{d}_2}{\text{D}} \quad \text{and} \quad h'' = \frac{t\text{d}_1}{\text{D}}$$

(2) *Thick Lens*—

$$f_1'' = -\mu f_1' = \frac{-\mu}{\text{d}_1}$$

$$f_2' = \frac{\mu}{d_2}, \quad f_2'' = \frac{-1}{d_2} \text{ and } \frac{1}{K} = \frac{-d_1 d_2}{\mu D}$$

$$D = d_1 + d_2 - \frac{t}{\mu} d_1 d_2, \quad h' = \frac{-td_2}{\mu D}, \quad h'' = \frac{td_1}{\mu D}$$

(2′) *Back focal distance* = F″B, where B denotes the back surface of the lens, so the *back focal power* D_b of the lens is $\frac{-1}{F''B} = \frac{-1}{F''H'' + H''B}$

$$D_b = -\frac{t + f_1'' - f_2'}{f_2''(f'' + t)} = \frac{-1}{f_2''} + \frac{f_2'}{f_2''(f_1'' + t)}$$

$$D_b = d_2 + \frac{\mu d_1}{\mu - td_1} \text{ or } d_1 = \frac{\mu (D_b - d_2)}{\mu + (D_b - d_2)} t$$

(3) *Lens in air before eye*

where $d_2 = +58{\cdot}636$ D and $f_2'' = \frac{-\mu}{d_2}$

$$D = d_1 + d_2 - td_1 d_2, \; h' = \frac{-td_2}{D}, \; h'' = \frac{\mu t d_1}{D}$$

Tilted Thin Lenses when incident rays are centric, parallel and oblique at angle ϕ.

$$D_c = D_2 \tan^2 \phi, \qquad D_2 = \frac{D}{\mu - 1} \; \frac{\sin(\phi - \phi')}{\sin \phi'}$$

Decentration and Centrads.—

If N denote the number of centrads and l the decentration in millimetres,

$$l = \frac{10\,N}{D} \quad \text{and } N = \frac{lD}{10}$$

If the lens be a spherocylinder, and if the cylinder be placed at axis θ, the value of D in the above formula must be the sum of the spherical component and the cylindrical component in either the vertical or horizontal meridian.

In the vertical meridian $C_\theta = C \cos^2\theta$.

In the horizontal meridian $C_\theta = C \sin^2\theta$.

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ and $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta.)$

The formulæ for Eccentric Pencils in B will hold for refraction at a plane surface by putting $r = \infty$.

$$v_2 = \frac{\mu'}{\mu_0} u \quad \text{and} \quad v_1 = \frac{\mu' \cos^2\phi'}{\mu_0 \cos^2\phi} u.$$

All the formulæ in B can be converted into the corresponding formulæ for reflection by putting $\mu' = -1$, and $\mu_0 = 1$.

Then $\phi = -\phi'$ and $f' = f''$

Table I. Metric Reciprocals.

This table gives the values in centimetres and inches of some reciprocals of a metre. Thus, the focal length of a given dioptric strength is given, as well as the slanting distance M O (*Fig.* 9, *p.* 115) of the object from the centre of motility when the number of metre angles is given.

E.g., a 2·25 D lens has as its focal length 44·4 cm. or $17\frac{1}{2}$ ins. ; and this is also the distance M O when the convergence is 2·25 m.a.

TABLE I.

D	Cm.	In.	D	Cm.	In.
·25	400	157·48	5	20	7·87
·5	200	78·74	5·5	$18\cdot\dot{1}\dot{8}$	7·158
·75	$133\cdot\dot{3}$	52·49	6	$16\cdot\dot{6}$	6·562
1·	100	39·37	7	14·286	5·624
1·25	80	31·496	8	12·5	4·921
1·5	$66\cdot\dot{6}$	26·247	9	$11\cdot\dot{1}$	4·375
1·75	57·143	22·497	10	10	3·937
2	50	19·685	11	$9\cdot\dot{0}\dot{9}$	3·579
2·25	$44\cdot\dot{4}$	17·499	12	$8\cdot\dot{3}$	3·281
2·5	40	15·748	13	7·692	3·028
2·75	$36\cdot\dot{3}\dot{6}$	14·316	14	7·143	2·812
3	$33\cdot\dot{3}$	13·124	15	$6\cdot\dot{6}$	2·625
3·5	28·57	11·25	16	6·25	2·462
4	25	9·843	18	$5\cdot\dot{5}$	2·187
4·5	$22\cdot\dot{2}$	8·749	20	5	1·969

Table II. Metre Angles.

As explained on *p.* 85, the value of the unit metre angle of Nagel depends on half the interocular distance. If this be denoted by m millimetres, 1 m.a. $= \sin^{-1}\left(\frac{m}{1000}\right)$ The interocular distance (I. O.) may vary from 56 mm. to 64 mm., and the corresponding value of 1 m.a. is given below for some interocular distances.

I.O.	56 mm.	58 mm.	60 mm.	62 mm.	64 mm.
1 m.a.	1° 36·27′ 2·8004 ▽	1° 39·71′ 2·9004 ▽	1° 43·15′ 3·0005 ▽	1° 46·59′ 3·1006 ▽	1° 50·03′ 3·2006 ▽

There is a difficulty in multiplication, as 10 m.a., for instance, is not ten times the size of 1 m.a. In the subjoined table the value is given of some of the multiples of that metre angle which corresponds to the most common interocular distance, viz., 60 mm.

TABLE II.

M.A.	Degrees	Centrads	M.A.	Degrees	Centrads
1	1° 43·15′	3·000 ▽	9	15° 39·86′	27·339 ▽
2	3° 26·39′	6·004 ▽	10	17° 27·46′	30·469 ▽
3	5° 9·82′	9·012 ▽	11	19° 16·13′	33·630 ▽
4	6° 53·53′	12·029 ▽	12	21° 6·00′	36·827 ▽
5	8° 37·62′	15·057 ▽	13	22° 57·27′	40·063 ▽
6	10° 22·19	18·099 ▽	14	24° 50·08′	43·345 ▽
7	12° 7·34′	21·157 ▽	15	26° 44·62′	46·677 ▽
8	13° 53·19′	24·237 ▽	16	28° 41·12′	50·065 ▽

TABLE III.—TRIGONOMETRICAL RATIOS, ETC.

If the number of degrees is under 45, the names of the ratios are given at the top of the table. Thus sin 10° is ·17365, and tan 20° is ·36397.

If the number of degrees is over 45, the names of the ratios are given at the foot of the table. Thus sin 65° is ·90631 and cos 76° is ·24192. The number of centrads corresponding to each degree up to 90° is given in a separate column.

For intermediate values the rule of proportional parts may be used, e.g., find sin 20° 40′.

The difference for 60′ between 20° and 21° is ·01635

The difference for 40′ or $\frac{2}{3}$ of 60′ is ·01090

sin 20° = ·34202

sin 20° 40′ = ·35292

The values of the functions of angles above 90° can be easily read off from the following table for the case when $\alpha = 90° + \theta$, etc.

	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$
$90° + \theta$	$\cos \theta$	$-\sin \theta$	$-\cot \theta$
$180° + \theta$	$-\sin \theta$	$-\cos \theta$	$\tan \theta$
$270° + \theta$	$-\cos \theta$	$\sin \theta$	$-\cot \theta$

It is often necessary when a cylinder is set obliquely at an angle θ, to find its power in the vertical and in the horizontal meridian to three decimal places.

Vertical $C_\theta = C \cos^2\theta = C \frac{1}{2} (1 + \cos 2\theta)$

Horizontally $C_\theta = C \sin^2\theta = C \frac{1}{2} (1 - \cos 2\theta)$

TABLE III.

Angle		Sine	Cosine	Tangent	Cotangent	Secant	Cosecant		
Degrees	Centrads								
0°	0	0	1	0	∞	1	∞	157·080	90°
1°	1·745	·01745	·99985	·01746	57·2900	1·00015	57·2987	155·334	89°
2°	3·491	·03490	·99939	·03492	28·6363	1·00061	28·6537	153·589	88°
3°	5·236	·05234	·99863	·05241	19·0811	1·00137	19·1073	151·844	87°
4°	6·981	·06976	·99756	·06993	14·3007	1·00244	14·3356	150·098	86°
5°	8·727	·08716	·99619	·08749	11·4301	1·00382	11·4737	148·353	85°
6°	10·472	·10453	·99452	·10510	9·51436	1·00551	9·56677	146·608	84°
7°	12·217	·12187	·99255	·12278	8·14435	1·00751	8·20551	144·862	83°
8°	13·963	·13917	·99027	·14054	7·11537	1·00983	7·18530	143·117	82°
9°	15·708	·15643	·98769	·15838	6·31375	1·01247	6·39245	141·372	81°
10°	17·453	·17365	·98481	·17633	5·67128	1·01543	5·75877	139·626	80°
11°	19·199	·19081	·98163	·19438	5·14455	1·01872	5·24084	137·881	79°
12°	20·944	·20791	·97815	·21256	4·70463	1·02234	4·80973	136·136	78°
13°	22·689	·22495	·97437	·23087	4·33148	1·02630	4·44541	134·390	77°
14°	24·435	·24192	·97030	·24933	4·01078	1·03061	4·13357	132·645	76°
15°	26·180	·25882	·96593	·26795	3·73205	1·03528	3·86370	130·900	75°
16°	27·925	·27564	·96126	·28675	3·48741	1·04030	3·62796	129·154	74°
17°	29·671	·29237	·95630	·30573	3·27085	1·04569	3·42030	127·409	73°
18°	31·416	·30902	·95106	·32492	3·07768	1·05146	3·23607	125·664	72°
19°	33·161	·32557	·94552	·34433	2·90421	1·05762	3·07155	123·918	71°
20°	34·907	·34202	·93969	·36397	2·74748	1·06418	2·92380	122·173	70°
21°	36·652	·35837	·93358	·38386	2·60509	1·07115	2·79043	120·428	69°
22°	38·397	·37461	·92718	·40403	2·47509	1·07853	2·66947	118·682	68°
23°	40·143	·39073	·92050	·42447	2·35585	1·08636	2·55930	116·937	67°
24°	41·888	·40674	·91355	·44523	2·24604	1·09464	2·45859	115·192	66°
25°	43·633	·42262	·90631	·46631	2·14451	1·10338	2·36620	113·446	65°
26°	45·379	·43837	·89879	·48773	2·05030	1·11260	2·28117	111·701	64°
27°	47·124	·45399	·89101	·50953	1·96261	1·12233	2·20269	109·956	63°
28°	48·869	·46947	·88295	·53171	1·88073	1·13257	2·13005	108·210	62°
29°	50·615	·48481	·87462	·55431	1·80405	1·14335	2·06267	106·465	61°
30°	52·360	·50000	·86603	·57735	1·73205	1·15470	2·00000	104·720	60°
31°	54·105	·51504	·85717	·60086	1·66428	1·16663	1·94160	102·974	59°
32°	55·851	·52992	·84805	·62487	1·60033	1·17918	1·88708	101·229	58°
33°	57·596	·54464	·83867	·64941	1·53987	1·19236	1·83608	99·484	57°
34°	59·341	·55919	·82904	·67451	1·48256	1·20622	1·78829	97·738	56°
35°	61·087	·57358	·81915	·70021	1·42815	1·22077	1·74345	95·993	55°
36°	62·832	·58779	·80902	·72654	1·37638	1·23607	1·70130	94·248	54°
37°	64·577	·60182	·79864	·75355	1·32704	1·25214	1·66164	92·502	53°
38°	66·323	·61566	·78801	·78129	1·27994	1·26902	1·62427	90·757	52°
39°	68·068	·62932	·77715	·80978	1·23490	1·28676	1·58902	89·012	51°
40°	69·813	·64279	·76604	·83910	1·19175	1·30541	1·55572	87·266	50°
41°	71·559	·65606	·75471	·86929	1·15037	1·32501	1·52425	85·521	49°
42°	73·304	·66913	·74314	·90040	1·11061	1·34563	1·49448	83·776	48°
43°	75·049	·68200	·73135	·93252	1·07237	1·36733	1·46628	82·030	47°
44°	76·794	·69466	·71934	·96569	1·03553	1·39016	1·43956	80·285	46°
45°	78·540	·70711	·70711	1·00000	1·00000	1·41421	1·41421	78·540	45°
		Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	Centrads	Degrees
								Angles	

TABLE IV.—SQUARES AND RECIPROCALS OF NUMBERS FROM 1 TO 99.

TABLE IV.

n	n^2	$\frac{1}{n}$	n	n^2	$\frac{1}{n}$	n	n^2	$\frac{1}{n}$
1	1	1	**34**	1156	·029412	**67**	4489	·014925
2	4	·5	**35**	1225	·028571	**68**	4624	·014706
3	9	·333333	**36**	1296	·027778	**69**	4761	·014493
4	16	·25	**37**	1369	·027027	**70**	4900	·014286
5	25	·2	**38**	1444	·026316	**71**	5041	·014085
6	36	·166667	**39**	1521	·025641	**72**	5184	·013889
7	49	·142857	**40**	1600	·025	**73**	5329	·013699
8	64	·125	**41**	1681	·024390	**74**	5476	·013514
9	81	·111111	**42**	1764	·023810	**75**	5625	·013333
10	100	·1	**43**	1849	·023256	**76**	5776	·013158
11	121	·090909	**44**	1936	·022727	**77**	5929	·012987
12	144	·083333	**45**	2025	·022222	**78**	6084	·012821
13	169	·076923	**46**	2116	·021739	**79**	6241	·012658
14	196	·071429	**47**	2209	·021277	**80**	6400	·0125
15	225	·066667	**48**	2304	·020833	**81**	6561	·012346
16	256	·0625	**49**	2401	·020408	**82**	6724	·012195
17	289	·058824	**50**	2500	·02	**83**	6889	·012048
18	324	·055556	**51**	2601	·019608	**84**	7056	·011905
19	361	·052632	**52**	2704	·019231	**85**	7225	·011765
20	400	·05	**53**	2809	·018868	**86**	7396	·011628
21	441	·047619	**54**	2916	·018519	**87**	7569	·011494
22	484	·045455	**55**	3025	·018182	**88**	7744	·011364
23	529	·043478	**56**	3136	·017857	**89**	7921	·011236
24	576	·041667	**57**	3249	·017544	**90**	8100	·011111
25	625	·04	**58**	3364	·017241	**91**	8281	·010989
26	676	·038462	**59**	3481	·016949	**92**	8464	·010870
27	729	·037037	**60**	3600	·016667	**93**	8649	·010753
28	784	·035714	**61**	3721	·016393	**94**	8836	·010638
29	841	·034483	**62**	3844	·016129	**95**	9025	·010526
30	900	·033333	**63**	3969	·015873	**96**	9216	·010417
31	961	·032258	**64**	4096	·015625	**97**	9409	·010309
32	1024	·03125	**65**	4225	·015385	**98**	9604	·010204
33	1089	·030303	**66**	4356	·015152	**99**	9801	·010101

INDEX.

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